Neap

Trial Examination 2023

HSC Year 12 Mathematics Advanced

Solutions and Marking Guidelines

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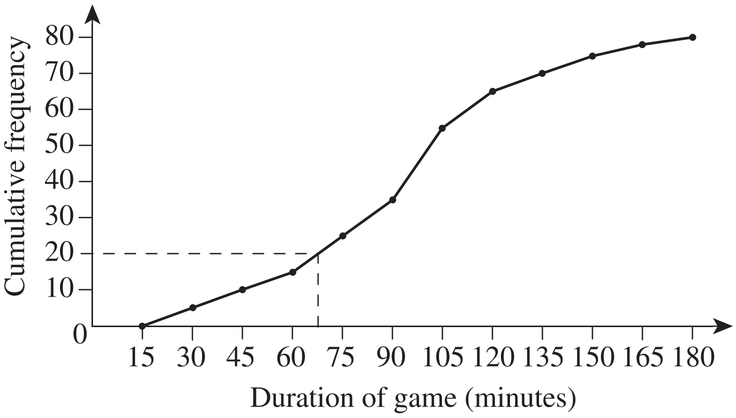
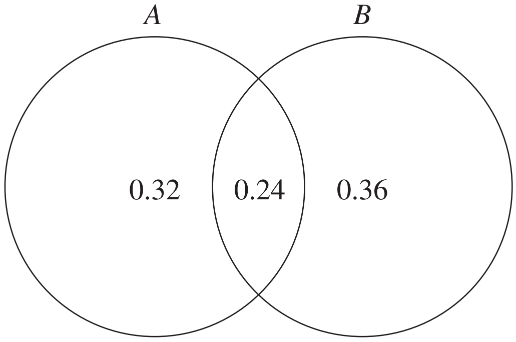
SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **D** $m = \tan\theta$ $= \tan\left(\pi - \frac{2\pi}{3}\right)$ $= \tan\left(\frac{\pi}{3}\right)$ $= \sqrt{3}$	MA–C1 Introduction to Differentiation MA11–1 Bands 3–4
Question 2 **C** Case 1: $2x - 1 = 5$ $2x = 6$ $x = 3$ Case 2: $-(2x - 1) = 5$ $2x - 1 = -5$ $2x = -4$ $x = -2$ $\therefore x = -2, 3$	MA–F1 Working with Functions MA11–1, 11–2 Band 3
Question 3 **C** $\text{pH} = -\log_{10}[\text{H}^+]$ $1.5 = -\log_{10}[\text{H}^+]$ $-1.5 = \log_{10}[\text{H}^+]$ $10^{-1.5} = [\text{H}^+]$	MA–E1 Logarithms and Exponentials MA11–6 Bands 3–4
Question 4 **D** The parabola is concave up; hence, $a > 0$. The parabola has a y-intercept that is positive; hence, $c > 0$. The parabola does not have x-intercepts; hence, $b^2 - 4ac < 0$.	MA–F1 Working with Functions MA11–2, 11–9 Band 4

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 5 **B** Reading from the graph, there was a total of 80 games played in the tournament. 25% of 80 = 20 games Using the graph, 25% of the games took less than 67.5 minutes to finish.  	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–8, 12–10 Band 4
Question 6 **B** $y = \ln\sqrt{\dfrac{x+1}{x-1}}$ $= \ln\left(\dfrac{x+1}{x-1}\right)^{\frac{1}{2}}$ $= \ln(x+1)^{\frac{1}{2}} - \ln(x-1)^{\frac{1}{2}}$ $= \dfrac{1}{2}\ln(x+1) - \dfrac{1}{2}\ln(x-1)$ $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{1}{x+1}\right) - \dfrac{1}{2}\left(\dfrac{1}{x-1}\right)$ $= \dfrac{1}{2}\left(\dfrac{1}{x+1} - \dfrac{1}{x-1}\right)$	MA–C3 Applications of Differentiation MA12–3, 12–6 Bands 4–5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 **A** To form the function $g(x)$, $f(x)$ has been: • reflected about the y-axis, forming $f(-x)$ • reflected about the x-axis, forming $-f(-x)$ • translated 2 units to the left, forming $-f(-(x-2))$. Hence: $g(x) = -f(-(x-2))$ $= -f(2-x)$	MA–F2 Graphing Techniques MA12–1 Bands 4–5
Question 8 **A** The equations will have no real solutions if they are parallel to each other and have different y-intercepts. Rearranging the equations to find the gradients of each equation gives: $ax + y - 4 = 0$ $y = -ax + 4$ $\therefore m_1 = -a$ $x + 2y - a = 0$ $2y = -x + a$ $y = -\frac{x}{2} + \frac{a}{2}$ $\therefore m_2 = -\frac{1}{2}$ For both lines to be parallel, $m_1 = m_2$. Therefore: $-a = -\frac{1}{2}$ $a = \frac{1}{2}$	MA–F1 Working with Functions MA11–9 Band 5

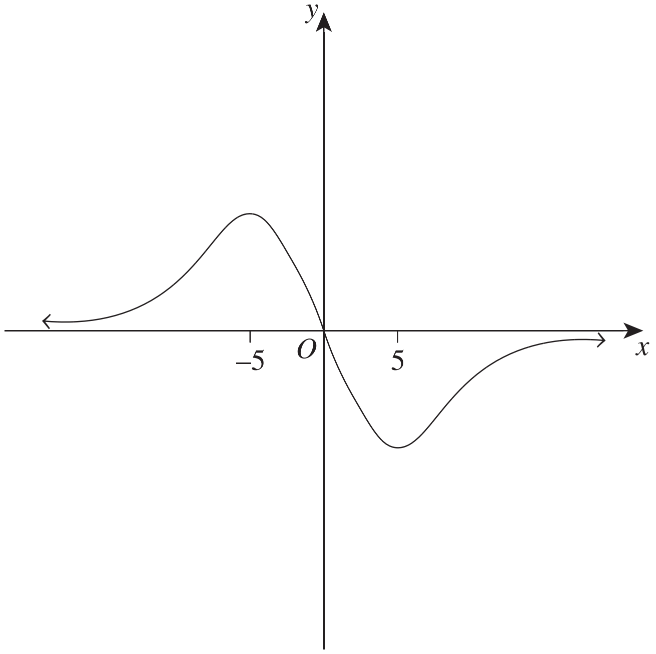
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **C** Using the information provided to find $P(A \cap B)$ gives: $P\left(A \middle\| B\right) = \frac{P(A \cap B)}{P(B)}$ $0.4 = \frac{P(A \cap B)}{0.6}$ $0.4 \times 0.6 = P(A \cap B)$ $P(A \cap B) = 0.24$ Using the information provided to find $P\left(A \cap \overline{B}\right)$ gives: $P\left(\overline{B}\right) = 1 - P(B)$ $= 1 - 0.6$ $= 0.4$ $P\left(A \middle\| \overline{B}\right) = \frac{P\left(A \cap \overline{B}\right)}{P\left(\overline{B}\right)}$ $0.8 = \frac{P\left(A \cap \overline{B}\right)}{0.4}$ $0.8 \times 0.4 = P\left(A \cap \overline{B}\right)$ $P\left(A \cap \overline{B}\right) = 0.32$ Thus, a complete Venn diagram can be constructed.  $P\left(B \middle\| A\right) = \frac{P(B \cap A)}{P(A)}$ $= \frac{0.24}{0.56}$ $= \frac{3}{7}$	MA–S1 Probability and Discrete Probability Distributions MA11–7, 11–9 Bands 5–6
Question 10 **A** The domain of $f(g(x))$ is (–1, 2] as it depends on the domain of $g(x)$. Reading from the graph, the range of $g(x)$ is [–1, 2]. Hence, $f(g(x))$ will only take on these values as inputs. According to the graph of $f(x)$, the range is [–3, 3] in the restricted domain [–1, 2]. Hence, the range of $f(g(x))$ is [–3, 3].	MA–F1 Working with Functions MA11–2, 11–9 Band 6

SECTION II

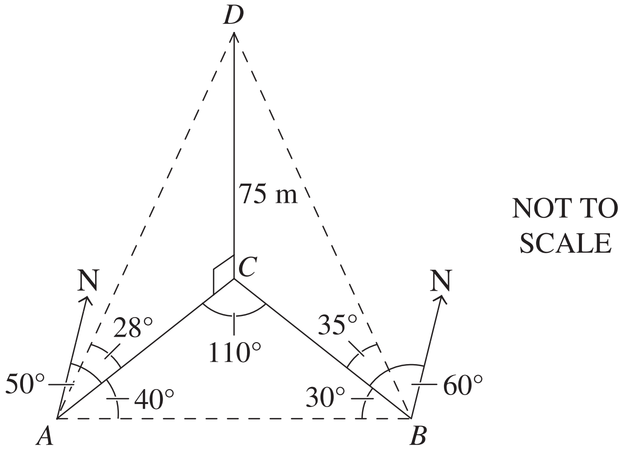
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
$9^{2x-3} = 27^x$ $3^{2(2x-3)} = 3^{3x}$ Equating the powers gives: $2(2x-3) = 3x$ $4x - 6 = 3x$ $4x - 3x = 6$ $x = 6$	MA–E1 Logarithms and Exponentials MA11–6 Bands 3–4 • Provides the correct solution . . . . . . 2 • Applies index laws. 1
Question 12	
Since $\sum p(x) = 1$: $0.35 + a + b + 0.15 + 0.05 + 0.01 = 1$ $a + b = 0.44 \quad (1)$ $E(X) = \Sigma xp(x)$ $= 0 \times 0.35 + 1 \times a + 2 \times b + 3 \times 0.15 + 4 \times 0.05 + 5 \times 0.01$ $= a + 2b + 0.7$ Since $E(X) = 1.5$: $1.5 = a + 2b + 0.7$ $a + 2b = 0.8 \quad (2)$ Subtracting (1) from (2) gives: $2b - b = 0.8 - 0.44$ $b = 0.36$ Substituting $b = 0.36$ into (1) gives: $a + 0.36 = 0.44$ $a = 0.08$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4 • Provides the correct solution . . . . . . 3 • Finds the value of *a* OR *b* . . . . 2 • Makes progress towards finding the values of *a* and *b* 1
Question 13	
(a) The number of squares in each figure forms an arithmetic sequence. F_1 has one square, so $a = 1$; the difference between F_1 and F_2 is 2, so $d = 2$. Therefore, the formula for the sequence is: $T_n = a + (n-1)d$ $F_n = 1 + (n-1) \times 2$ Substituting $n = 15$ gives: $F_{15} = 1 + (15-1) \times 2$ $= 29$	MA–M1 Modelling Financial Situations MA12–4, 12–10 Band 3 • Provides the correct solution . . . . . . 2 • Finds an expression for F_n. 1

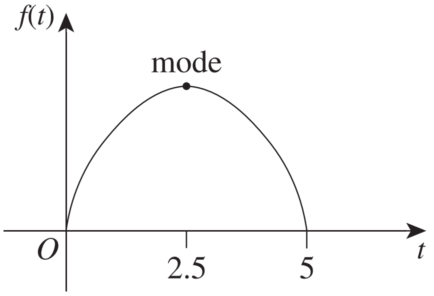
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) Finding the value of n when $F_n = 175$ gives: $F_n = a + (n-1)d$ $175 = 1 + (n-1) \times 2$ $174 = 2(n-1)$ $87 = n-1$ $n = 88$ Since n is an integer, it is possible to have a figure with 175 squares; F_{88} has 175 squares.	MA–M1 Modelling Financial Situations MA12–4, 12–10 Bands 3–4 • Provides the correct solution . . . . . . 2 • Makes progress towards solving an equation for n 1
(c) Finding S_{50} gives: $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ $S_{50} = \frac{50}{2}\left(2 \times 1 + (50-1) \times 2\right)$ $= 2500$ Therefore, 2500 squares are needed to make the first 50 figures.	MA–M1 Modelling Financial Situations MA12–4, 12–10 Bands 3–4 • Provides the correct solution . . . . . . 2 • Finds an expression for S_{50} 1
Question 14	
$y = \left(3x^2 + 1\right)^3$ $\frac{dy}{dx} = 3\left(3x^2 + 1\right)^2 \times 6x$ $= 18x\left(3x^2 + 1\right)^2$	MA–C2 Differential Calculus MA12–3, 12–6 Bands 3–4 • Provides the correct solution . . . . . . 2 • Makes progress towards finding the derivative. 1
Question 15	
$\int \frac{5x^3 - 2x}{x^5}\,dx = \int \frac{5x^3}{x^5} - \frac{2x}{x^5}\,dx$ $= \int \frac{5}{x^2} - \frac{2}{x^4}\,dx$ $= \int 5x^{-2} - 2x^{-4}\,dx$ $= \left[-\frac{5}{x} + \frac{2}{3x^3}\right] + C$	MA–C4 Integral Calculus MA12–7 Band 4 • Provides the correct solution . . . . . . 2 • Makes progress towards simplifying the integrand. 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
Question 16	
$\int_1^4 2\sqrt{x}+\frac{3}{x}dx=\int_1^4 2x^{\frac{1}{2}}+\frac{3}{x}dx$ $=\left[\frac{4}{3}x^{\frac{3}{2}}+3\ln x\right]_1^4$ $=\left(\frac{4}{3}\times 4^{\frac{3}{2}}+3\ln 4\right)-\left(\frac{4}{3}\times 1^{\frac{3}{2}}+3\ln 1\right)$ $=\frac{32}{3}+3\ln 4-\frac{4}{3}$ $=\frac{28}{3}+6\ln 2$	MA–C4 Integral Calculus MA12–7 Band 4 • Provides the correct solution . . . . . . 2 • Finds the anti-derivative 1
Question 17	
(a) $Q_1=67$ and $Q_3=79.5$ $IQR=79.5-67$ $=12.5$	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–8 Band 3 • Provides the correct solution . . . . . . 2 • Identifies Q_1 OR Q_3. 1
(b) Outliers are outside the upper and lower bounds of a data set. To be an outlier, the mark needs to be less than $Q_1 - 1.5 \times IQR$. $Q_1-1.5\times IQR=67-1.5\times 12.5$ $=48.25$ Therefore, the result of 35% is considered an outlier. *Note: Consequential on answer to **Question 17(a)**.*	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–8 Band 3 • Provides the correct solution 1
(c) The scatterplot indicates a positive correlation. Hence, the correlation coefficient cannot be negative. The correlation coefficient must be within $-1\le r\le 1$. Hence, –2.6577 is not an acceptable value.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–10 Bands 3–4 • Provides TWO correct reasons . . . . 2 • Provides ONE correct reason. 1
(d) The correlation between the students' school attendance and Mathematics test results is strong and positive.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–8 Band 3 • Describes the strength AND direction of the correlation . . . . 2 • Describes the strength OR direction of the correlation 1

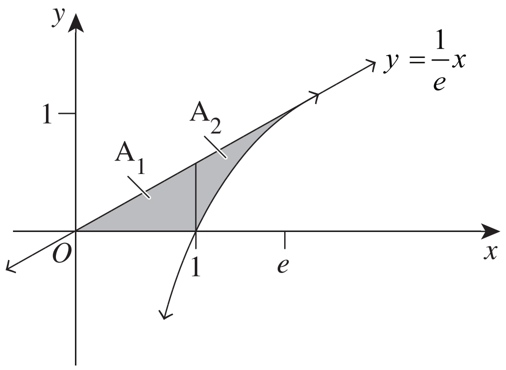
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) Substituting $x = 88$ into the equation of the least-squares regression line gives: $y = 0.7341x + 12.151$ $= 0.7341 \times 88 + 12.151$ $= 77\%$ (nearest percentage)	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–10 Band 3 • Provides the correct solution 1
Question 18	
	MA–C3 Applications of Differentiation MA12–6 Bands 3–4 • Sketches a graph that shows all THREE of: – an x-intercept at (0, 0) – turning points at $x = \pm 5$ – a horizontal asymptote at $y = 0$ . . . . 3 • Sketches a graph that shows any TWO of the above points . . . . . 2 • Sketches a graph that shows any ONE of the above points. 1
Question 19	
Rearranging $y = 5\sin\left(2x + \frac{\pi}{3}\right)$ to identify the transformations involved gives: $y = 5\sin\left(2x + \frac{\pi}{3}\right)$ $\frac{y}{5} = \sin\left(2\left(x + \frac{\pi}{6}\right)\right)$ $\frac{y}{5} = \sin\left(\frac{x - \left(-\frac{\pi}{6}\right)}{\frac{1}{2}}\right)$ Therefore, the correct order of transformations is: 1. a horizontal dilation with a scale factor of $\frac{1}{2}$ 2. a horizontal translation of $\frac{\pi}{6}$ units to the left 3. a vertical dilation with a scale factor of 5.	MA–T3 Trigonometric Functions and Graphs MA12–5, 12–10 Band 4 • Outlines all THREE graphical transformations in the correct order. . . . . 3 • Outlines TWO graphical transformations . . . . 2 • Outlines ONE graphical transformation 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 20	
The common ratio for the geometric sequence is: $r=\frac{-12}{6}$ $=-2$ However, negative numbers cannot be used with the natural logarithm. Therefore, using $r = 2$ to find the number of terms in the geometric sequence gives: $T_n = ar^{n-1}$ $1536 = 6\times 2^{n-1}$ $256 = 2^{n-1}$ $\ln 256 = \ln 2^{n-1}$ $\ln 256 = (n-1)\ln 2$ $\frac{\ln 256}{\ln 2} = n-1$ $n = 9$ Hence, finding the sum of the first nine terms using $r = -2$ gives: $S_n = \frac{a(r^n-1)}{r-1}$ $S_9 = \frac{6(-2^9-1)}{-2-1}$ $=1026$	MA–M1 Modelling Financial Situations MA12–4 Bands 4–5 • Provides the correct solution . . . . . . 4 • Makes progress towards finding the sum of the first nine terms in the geometric sequence . . . . 3 • Determines that there are nine terms in the geometric sequence . . . 2 • Provides the correct expression for the geometric sequence including the correct values for a and r 1
Question 21	
The interest factor for six years at a rate of 3% is 6.4684. The total amount at the end of six years is: $A_6 = 5000\times 6.4684$ $= 32\ 342$ In the seventh and eighth years, A_6 continues to earn interest and Duncan makes an additional \$5000 deposit for each year. Hence: $A_8 = 32\ 342\times 1.025^2 + 5000\times 1.025^2 + 5000\times 1.025$ $= \$44\ 357.44$	MA–M1 Modelling Financial Situations MA12–4, 12–10 Band 5 • Provides the correct solution . . . . . . 3 • Calculates the value of A_6. AND • Makes progress towards calculating the value of A_8. . . . . 2 • Calculates the value of A_6 1

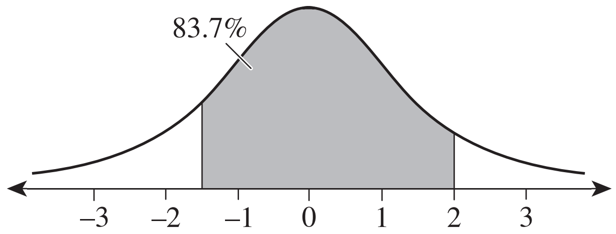
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 22	
Using the bearings to calculate $\angle CAB$ and $\angle CBA$ gives:  Given that $\angle CAB = 40°$ and $\angle CBA = 30°$, $\angle ACB = 110°$. Finding an expression for the distance between points A and C gives: $\tan 28° = \frac{75}{AC}$ $AC = \frac{75}{\tan 28°}$ Therefore, finding the distance between points A and B gives: $\frac{AB}{\sin 110°} = \frac{AC}{\sin 30°}$ $\frac{AB}{\sin 110°} = \frac{\left(\frac{75}{\tan 28°}\right)}{\sin 30°}$ $AB = \sin 110° \times \frac{\left(\frac{75}{\tan 28°}\right)}{\sin 30°}$ $= 265 \text{ m}$ *Note: Diagrams are not required to achieve full marks, but may be used to develop the response.*	MA–T1 Trigonometry and Measure of Angles MA11–1, 11–9 Bands 4–5 • Provides the correct solution . . . . . . 3 • Makes progress towards finding *AB* by applying the sine rule OR equivalent merit . . . . 2 • Finds an expression for AC 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 23	
(a) Since $\int_{-\infty}^{\infty} f(t)\,dt = 1$: $\int_0^5 kt(5-t)\,dt = 1$ $\int_0^5 5t - t^2\,dt = \frac{1}{k}$ $\left[\frac{5t^2}{2} - \frac{t^3}{3}\right]_0^5 = \frac{1}{k}$ $\left(\frac{5\times5^2}{2} - \frac{5^3}{3}\right) = \frac{1}{k}$ $\frac{125}{6} = \frac{1}{k}$ $k = \frac{6}{125}$	MA–S3 Random Variables MA12–8, 12–10 Bands 3–4 • Provides the correct solution . . . . . . 2 • Finds the anti-derivative of $f(t)$ 1
(b) The mode is the value of t that gives the maximum value of $f(t)$. Since $f(t) = \frac{6}{125}t(5-t)$ is concave down, the maximum value occurs at the axis of symmetry. Substituting $f(t) = 0$ to find the roots of the parabola gives: $\frac{6}{125}t(5-t) = 0$ $t = 0, 5$ Given that the axis of symmetry is the midpoint between these values:  The axis of symmetry is at $t = 2.5$ and, hence, the mode is $t = 2.5$. *Note: Diagrams are not required to achieve full marks, but may be used to develop the response.*	MA–S3 Random Variables MA12–8 Bands 3–4 • Provides the correct solution . . . . . . 2 • Makes progress towards finding the axis of symmetry of $f(t)$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $P(t \le 1) = \int_0^1 \frac{6}{125} t(5-t)dt$ $= \frac{6}{125}\int_0^1 5t - t^2 dt$ $= \frac{6}{125}\left[\frac{5t^2}{2} - \frac{t^3}{3}\right]_0^1$ $= \frac{6}{125}\left[\left(\frac{5}{2} - \frac{1}{3}\right) - (0-0)\right]$ $= \frac{13}{125}$ $= 0.104$ Therefore, 10.4% of the high-rise buildings are constructed within a year.	MA–S3 Random Variables MA12–8, 12–10 Bands 3–4 • Provides the correct solution . . . . . . 2 • Develops an integral to describe $P(T \le 1)$ 1
Question 24	
(a) $\frac{d}{dx}(x \ln x - x) = \ln x \times 1 + x \times \frac{1}{x} - 1$ $= \ln x + 1 - 1$ $= \ln x$	MA–C3 Applications of Differentiation MA12–3, 12–6 Bands 3–4 • Provides the correct solution . . . . . . 2 • Provides some relevant working . . . 1
(b) If $y = \ln x,\ y' = \frac{1}{x}$. When $x = e,\ y' = \frac{1}{e}$. Thus, the gradient of the tangent is $\frac{1}{e}$. Finding the equation of the tangent gives: $y - 1 = \frac{1}{e}(x - e)$ $e(y-1) = x - e$ $ey - e = x - e$ $ey = x$ $y = \frac{1}{e}x$	MA–C3 Applications of Differentiation MA12–3 Band 4 • Provides the correct solution . . . . . . 2 • Finds the gradient of the tangent . . . 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) Dividing the shaded area into two regions, A_1 and A_2, gives:  Given that A_1 is a triangle with a length of 1 and a height of $\frac{1}{e}$, finding the area of A_1 gives: $A_1 = \int_0^1 \frac{1}{e} x\,dx$ $= \frac{1}{2} \times 1 \times \frac{1}{e}$ $= \frac{1}{2e}$ Finding the area of A_2 gives: $A_2 = \int_1^e \frac{1}{e} x - \ln x\,dx$ $= \int_1^e \frac{1}{e} x dx - \int_1^e \ln x\,dx$ $= \int_1^e \frac{1}{e} x dx - [x \ln x - x]_1^e$ $= \frac{1}{e}\left[\frac{x^2}{2}\right]_1^e - [(e \ln e - e) - (0 - 1)]$ $= \frac{1}{e}\left[\frac{e^2}{2} - \frac{1}{2}\right] - 1$ $= \frac{e^2 - 1}{2e} - 1$ Therefore: $A_{\text{shaded}} = A_1 + A_2$ $= \frac{1}{2e} + \frac{e^2 - 1}{2e} - 1$ *Note: Consequential on answer to* ***Question 24(a)***.	MA–C4 Integral Calculus MA12–7, 12–10 Bands 4–5 • Provides the correct solution . . . . . . 3 • Finds the area of A_1. AND • Makes progress towards finding the area of A_2. . . . . 2 • Finds the area of A_1. OR • Provides some relevant working . . . 1

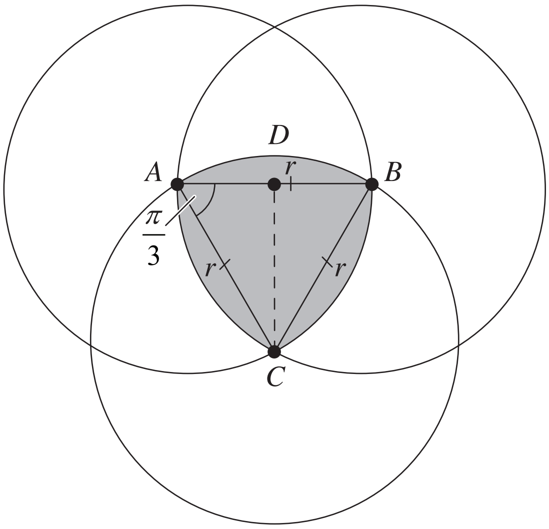
<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>Question 25</td><td></td></tr>
<tr><td>(a) 120 km/h is two standard deviations to the right of the mean.
Therefore, the percentage of drivers travelling at a dangerous speed is $\frac{5\%}{2} = 2.5\%,$ according to the empirical rule for normally distributed random variables.</td><td>MA–S3 Random Variables
MA12–8 Bands 3–4
• Provides the correct solution . . . . . . 1</td></tr>
<tr><td>(b) Finding the z-score for 102.5 km/h gives:
$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{102.5-110}{5} \\ &= -1.5 \end{aligned}$$
Finding the z-score for 120 km/h gives:
$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{120-110}{5} \\ &= 2 \end{aligned}$$
Therefore:
$$\begin{aligned} P(102.5 \le X \le 120) &= P(-1.5 \le z \le 2) \\ &= \int_{-1.5}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \end{aligned}$$</td><td>MA–S3 Random Variables
MA12–10 Band 4
• Provides the correct solution 2
———
• Finds the z-score for 102.5 km/h OR 120 km/h . . . . 1</td></tr>
<tr><td>(c) $$\begin{aligned} P(-1.5 \le z \le 2) &= \int_{-1.5}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-1.5}^{2} e^{-\frac{x^2}{2}}\, dx \end{aligned}$$
Thus, using $e^{-\frac{x^2}{2}}$ to generate a table of values gives:
<table>
<tr><td>x</td><td>–1.5</td><td>0.25</td><td>2</td></tr>
<tr><td>$f(x)$</td><td>0.325</td><td>0.969</td><td>0.135</td></tr>
</table>
Applying the trapezoidal rule gives:
$$\begin{aligned} P(-1.5 \le z \le 2) &= \frac{1}{\sqrt{2\pi}} \int_{-1.5}^{2} e^{-\frac{x^2}{2}}\, dx \\ &= \frac{1}{\sqrt{2\pi}} \left[\frac{2 - -1.5}{2 \times 2}(0.325 + 0.1353 + 2 \times 0.969) \right] \\ &= 0.837 \end{aligned}$$</td><td>MA–C4 Integral Calculus
MA12–3 Bands 3–4
• Provides the correct solution 2
———
• Provides the correct table of values. . . . . 1</td></tr>
</table>

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) From part (c): $P(-1.5 \le z \le 2) = 0.837$ $= 83.7\%$  Given that $P(-2 \le z \le 2) = 95\%$: $P(-2 \le z \le -1.5) = 95 - 83.7$ $= 11.3\%$ According to the empirical rule, $P(z \le -2) = 2.5\%$. Hence: $P(z \le -1.5) = 2.5 + 11.3$ $= 13.8\%$ Therefore, the probability of a driver travelling faster than 102.5 km/h is 100 – 13.8 = 86.2%. *Note: Consequential on answer to* ***Question 25(c)***.	MA–S3 Random Variables MA12–10 Bands 5–6 • Provides the correct solution . . . . . . 2 • Makes progress towards the correct solution 1
Question 26	
$\int_0^k e^x\left(e^x - 2\right)dx = \frac{3}{2}$ $\int_0^k e^{2x} - 2e^x\,dx = \frac{3}{2}$ $\left[\frac{1}{2}e^{2x} - 2e^x\right]_0^k = \frac{3}{2}$ $\left(\frac{1}{2}e^{2k} - 2e^k\right) - \left(\frac{1}{2} - 2\right) = \frac{3}{2}$ $\frac{1}{2}e^{2k} - 2e^k = 0$ $e^{2k} - 4e^k = 0$ $e^k\left(e^k - 4\right) = 0$ $e^k = 0,\ 4$ Given that $e^k > 0$ for all k, there are no real solutions from $e^k = 0$. Therefore: $e^k = 4$ $\ln e^k = \ln 4$ $k \ln e = \ln 4$ $k = \ln 4$ $= 2\ln 2$	MA–C4 Integral Calculus MA12–7, 12–10 Bands 4–5 • Provides the correct solution . . . . . . 4 • Identifies that there are no real solutions for $e^k = 0$. AND • Makes progress towards solving $e^k = 4$ . . . . 3 • Makes progress towards finding the solutions for e^k . . . . 2 • Finds the anti-derivative 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
Question 27	
(a) When $t = 0$: $x(t) = 5t$ $x(0) = 5(0)$ $= 0$ Therefore, the particle begins at the origin.	MA–C3 Applications of Differentiation MA12–3, 12–10 Bands 3–4 • Provides the correct solution 1
(b) For $0 \le t \le 1$: $x(t) = 5t$ $v(t) = \frac{dx}{dt}$ $= 5$ Given that $v > 0$ for $0 \le t \le 1$, the particle is never at rest during this time period. For $t > 1$: $x(t) = 6\sqrt{t} - \frac{1}{t}$ $= 6x^{\frac{1}{2}} - t^{-1}$ $v(t) = \frac{dx}{dt}$ $= 3t^{-\frac{1}{2}} + t^{-2}$ $= \frac{3}{\sqrt{t}} + \frac{1}{t^2}$ Since $v > 0$ for $t > 1$, the particle is never at rest during this time period. Hence, the particle is never at rest.	MA–C3 Applications of Differentiation MA12–3, 12–10 Bands 3–4 • Provides the correct solution . . . . . . 3 • Shows that the particle is never at rest for ONE of the time intervals . . . . 2 • Finds the velocity function for ONE of the time intervals 1
(c) In part (b), it is shown that the particle will always have a velocity such that $v > 0$. This means that the particle starts at the origin and will always move in the positive direction. Hence, the distance travelled in the first four seconds is given by: $x(t) = 6\sqrt{t} - \frac{1}{t}$ $x(4) = 6\sqrt{4} - \frac{1}{4}$ $= 11.75\text{ m}$	MA–C3 Applications of Differentiation MA12–3, 12–10 Bands 4–5 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
(a) $P(\text{Edie wins}) = \frac{1}{4}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 3 • Provides the correct solution 1
(b) If Edie is to win in the second round, both Edie and Catriona must lose in the first round. Therefore: $P(\text{E wins 2nd}) = P(\text{E loses 1st, C loses 1st, E wins 2nd})$ $= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{5}$ $= \left(\frac{3}{4}\right)^2 \times \frac{3}{5}$ Therefore, the probability that Edie wins in the first OR second round is: $P(\text{E wins 1st OR 2nd}) = P(\text{E wins 1st}) + P(\text{E wins 2nd})$ $= \frac{1}{4} + \left(\left(\frac{3}{4}\right)^2 \times \frac{3}{5}\right)$ *Note: Consequential on answer to **Question 28(a)**.*	MA–S1 Probability and Discrete Probability Distributions MA11–7, 11–9 Band 4 • Provides the correct solution . . . . . . 2 • Makes progress towards calculating the probability of Edie winning in the second round 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
(c) The series can be expressed as two separate geometric sequences, S_1 and S_2. $S_1 = \frac{1}{4} + \left(\left(\frac{3}{4}\right)^2 \times \left(\frac{3}{5}\right)^2 \times \frac{1}{4}\right) + \ldots$ $S_2 = \left(\left(\frac{3}{4}\right)^2 \times \frac{3}{5}\right) + \left(\left(\frac{3}{4}\right)^2 \times \left(\frac{3}{5}\right)^2 \times \frac{3}{5}\right) + \ldots$ Finding the limiting sum of S_1 gives: $S_1 = \frac{a}{1-r}$ $= \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^2 \times \left(\frac{2}{5}\right)^2}$ $= \frac{25}{91}$ Finding the limiting sum of S_2 gives: $S_2 = \frac{a}{1-r}$ $= \frac{\left(\frac{3}{4}\right)^2 \times \frac{3}{5}}{1-\left(\frac{3}{4}\right)^2 \times \left(\frac{2}{5}\right)^2}$ $= \frac{135}{364}$ Therefore: $\text{total probability} = S_1 + S_2$ $= \frac{25}{91} + \frac{135}{364}$ $= \frac{235}{364}$ Since $\frac{235}{364} > \frac{1}{2}$, Edie has a greater chance at winning than Catriona.	MA–M1 Modelling Financial Situations MA12–4, 12–10 Band 5–6 • Provides the correct solution . . . . . . 3 • Calculates the limiting sum of S_1 AND S_2 . . . . 2 • Calculates the limiting sum of S_1 OR S_2 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
Question 29	
$AB = AC = BC = r$, since they are all radii. Hence, ΔABC is an equilateral triangle and $\angle CAB = \frac{\pi}{3}$.  $A_{\Delta ABC} = \frac{1}{2} \times r^2 \sin\left(\frac{\pi}{3}\right)$ $= \frac{\sqrt{3}}{4} r^2$ Finding the area of one of the sections outside ΔABC gives: $A_{\text{segment outside } \Delta ABC} = A_{\text{sector of circle}} - A_{\Delta ABC}$ $= \frac{1}{2} r^2 \theta - \frac{\sqrt{3}}{4} r^2$ $= \frac{1}{2} \times r^2 \times \frac{\pi}{3} - \frac{\sqrt{3}}{4} r^2$ $= \frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2$ $A_{\text{shaded}} = 3 \times A_{\text{segment outside } \Delta ABC} + A_{\Delta ABC}$ $= 3\left(\frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2\right) + \frac{\sqrt{3}}{4} r^2$ $= \frac{\pi}{2} r^2 - \frac{\sqrt{3}}{2} r^2$ $= \frac{r^2}{2}\left(\pi - \sqrt{3}\right)$ $= \frac{1}{2} r^2 \left(\pi - \sqrt{3}\right)$ *Note: Diagrams are not required to achieve full marks, but may be used to develop the response.*	MA–T1 Trigonometry and Measure of Angles MA11–3, 11–9 Bands 5–6 • Provides the correct solution . . . . . . 4 • Finds an expression for the area of one segment outside ΔABC . . . . . 3 • Finds an expression for the area of a sector of a circle OR equivalent merit. . . . . 2 • Finds an expression for the area of ΔABC OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 30	
(a) Given that *BCD* is a triangle: $\tan\theta = \frac{b}{BD}$ $BD = \frac{b}{\tan\theta}$ $= b\cot\theta$ $\therefore AB = a - b\cot\theta$ $\sin\theta = \frac{b}{BC}$ $BC = b\text{cosec}\theta$ Therefore: $R = R_{AB} + R_{BC}$ $= k\,\frac{AB}{(r_1)^4} + k\,\frac{BC}{(r_2)^4}$ $= k\left(\frac{a - b\cot\theta}{(r_1)^4} + \frac{b\text{cosec}\theta}{(r_2)^4}\right)$	MA–T2 Trigonometric Functions and Identities MA11–1, 11–9 Band 5 • Provides the correct solution . . . . . . 2 • Finds an expression for *AB* OR *BC* 1
(b) $\frac{d}{dx}(\cot\theta) = \frac{d}{dx}\left(\frac{\cos\theta}{\sin\theta}\right)$ $= \frac{\sin\theta \times -\sin\theta - \cos\theta \times \cos\theta}{\sin^2\theta}$ $= \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}$ $= \frac{-(\sin^2\theta + \cos^2\theta)}{\sin^2\theta}$ $= -\frac{1}{\sin^2\theta}$ $= -\text{cosec}^2\theta$	MA–C2 Differential Calculus MA12–3 Bands 4–5 • Provides the correct solution . . . . . . 2 • Differentiates $\cot\theta$ using the quotient rule 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $R = k\left(\frac{a - b\cot\theta}{(r_1)^4} + \frac{b\operatorname{cosec}\theta}{(r_2)^4}\right)$ $\frac{dR}{d\theta} = k\left(\frac{b\operatorname{cosec}^2\theta}{(r_1)^4} - \frac{b\cot\theta\operatorname{cosec}\theta}{(r_2)^4}\right)$ $= bk\left(\frac{\operatorname{cosec}^2\theta}{(r_1)^4} - \frac{\cot\theta\operatorname{cosec}\theta}{(r_2)^4}\right)$ $= bk\left(\frac{(r_2)^4\operatorname{cosec}^2\theta - (r_1)^4\cot\theta\operatorname{cosec}\theta}{(r_1r_2)^4}\right)$ $= \frac{bk}{(r_1r_2)}\operatorname{cosec}\theta\left((r_2)^4\operatorname{cosec}\theta - (r_1)^4\cot\theta\right)$ Finding the stationary points by solving $\frac{dR}{d\theta} = 0$ gives: $\frac{bk}{(r_1r_2)}\operatorname{cosec}\theta\left((r_2)^4\operatorname{cosec}\theta - (r_1)^4\cot\theta\right) = 0$ $(r_2)^4\operatorname{cosec}\theta - (r_1)^4\cot\theta = 0$ $\left(\frac{r_2}{r_1}\right)^4 = \frac{\cot\theta}{\operatorname{cosec}\theta}$ $\left(\frac{r_2}{r_1}\right)^4 = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$ $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$ Given that $\frac{d^2R}{d\theta^2} > 0$ when $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$, $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$ is a minimum turning point. Hence, the resistance of the blood is minimised when $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$.	MA–C3 Applications of Differentiation MA12–6, 12–10 Band 6 • Provides the correct solution. AND • Justifies that the resistance of the blood is minimised when $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$ . . . . 4 • Shows that a stationary point occurs when $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$ . . . . . . 3 • Makes progress towards solving $\frac{dR}{d\theta} = 0$ . . . . 2 • Makes progress towards finding an expression for $\frac{dR}{d\theta}$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) When $r_2 = \frac{3}{4} r_1$: $\cos\theta = \left(\frac{r_2}{r_1}\right)^4$ $= \left(\frac{\frac{3}{4} r_1}{r_1}\right)^4$ $= \left(\frac{3}{4}\right)^4$ $\theta = \cos^{-1}\left(\left(\frac{3}{4}\right)^4\right)$ $= 72°$	MA–T2 Trigonometric Functions and Identities MA11–4 Band 4 • Provides the correct solution . . . . . . 2 • Provides some relevant working . . . 1