Neap

Final Examination 2023

NSW Year 11 Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 80

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 70 marks (pages 7–30)

- Attempt Questions 11–31
- Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The value of $\dfrac{23^{\pi} \times 4.1}{2023 \times \sqrt{e}}$, correct to three significant figures, is

A. 23.3

B. 23.315

C. 63.377

D. 63.4

2 A and B are two events such that $P(A) = a$, $P(B) = 2a$, $P(A \cap B) = 0.1$ and $P(A \cup B) = 0.5$. What is the value of a?

A. 0.1

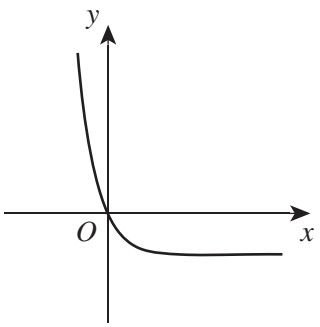
B. 0.2

C. 0.5

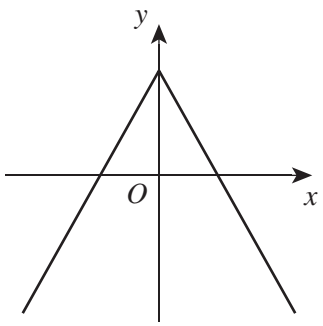
D. 0.6

3 Which of the following graphs represents an odd function?

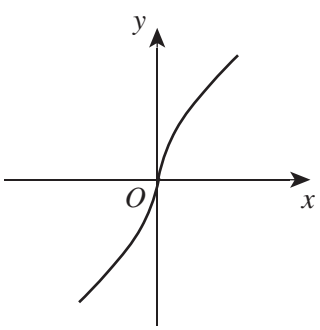
A.



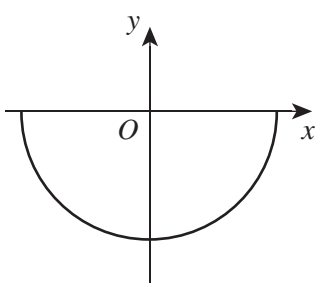
B.



C.



D.



4 Which of the following is equal to $\dfrac{\ln 7}{\ln 4}$?

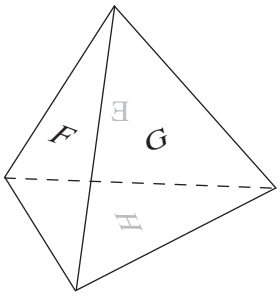
A. $\dfrac{\log_7 10}{\log_4 10}$

B. $10^{\frac{7}{4}}$

C. $\ln 7 - \ln 4$

D. $\log_4 7$

5 The diagram shows a fair four-sided die. The die has one letter inscribed on each of its four faces: E, F, G and H. When the die is rolled on a table, three of the die's faces are visible and the face that lies against the table is not visible.



The die is rolled once.

What is the probability that the visible letters are either E, F and H OR E, G and H?

A. $\dfrac{1}{16}$

B. $\dfrac{1}{2}$

C. $\dfrac{9}{16}$

D. $\dfrac{3}{4}$

6 Which of the following is the value of a that causes the parabola $y = -5x^2 + 7x - a$ to have two x-intercepts?

A. $\sin(\pi)$

B. 2.5

C. π

D. $\ln(e^{23})$

7 During a COVID-19 lockdown, a household of five people had enough toilet paper to last 28 days. At the beginning of these 28 days, two people returned from overseas. The same amount of toilet paper was used each day.

The relationship between the number of people in the household, P, and the number of days the toilet paper lasted, D, can be modelled by the equation $P = \frac{k}{D}$.

How many fewer days did the toilet paper last once the number of people in the household increased when the two people returned from overseas?

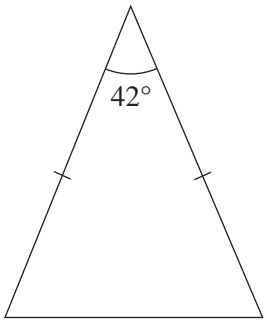
A. 8

B. 14

C. 20

D. 140

8 The diagram shows an isosceles triangle with an area of 42 cm^2 and equal sides of length x cm. The angle between the equal sides is 42°.



NOT TO SCALE

Which of the following expressions represents the length of the equal sides?

A. $\frac{x}{\sin(69°)} = \frac{42}{\sin(42°)}$

B. $42^2 = x^2 + x^2 - 2x^2 \times \cos(42°)$

C. $\frac{x^2 \times \cos(48°)}{2} = 42$

D. $x = \frac{84}{\sin\left(\frac{42\pi}{180}\right)}$

9 Consider the equations for A and B, where a and b are real numbers.

$$A = -a^2 + 8a + 1$$
$$B = b^2 + 18b + 5$$

What is the sum of the largest possible value of A and the smallest possible value of B?

A. –59

B. –51

C. 23

D. 67

10 The derivative of $f\left(x^3\right) \times g(x+1)$ is

A. $3x^2 f'\left(x^3\right) g(x+1) + 3(x+1)^2 f\left(x^3\right) g'(x+1)$

B. $3x^2 f'\left(x^3\right) g(x+1) + f\left(x^3\right) g'(x+1)$

C. $3x^2 f\left(x^3\right) g'(x+1) + f'\left(x^3\right) g(x+1)$

D. $3x^2 f'(x) g(x+1) + f\left(x^3\right) g'(x)$

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NSW Year 11 Mathematics Advanced

Section II Answer Booklet

70 marks
Attempt Questions 11–31
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve $18x = 9x^2$. **2**

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Question 12 (2 marks)

Expand and simplify $(3\sqrt{5} - \sqrt{3})(5\sqrt{3} + \sqrt{5})$. **2**

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Question 13 (2 marks)

Solve $\left|\frac{6-5x}{2}\right| = 3$. **2**

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Question 14 (2 marks)

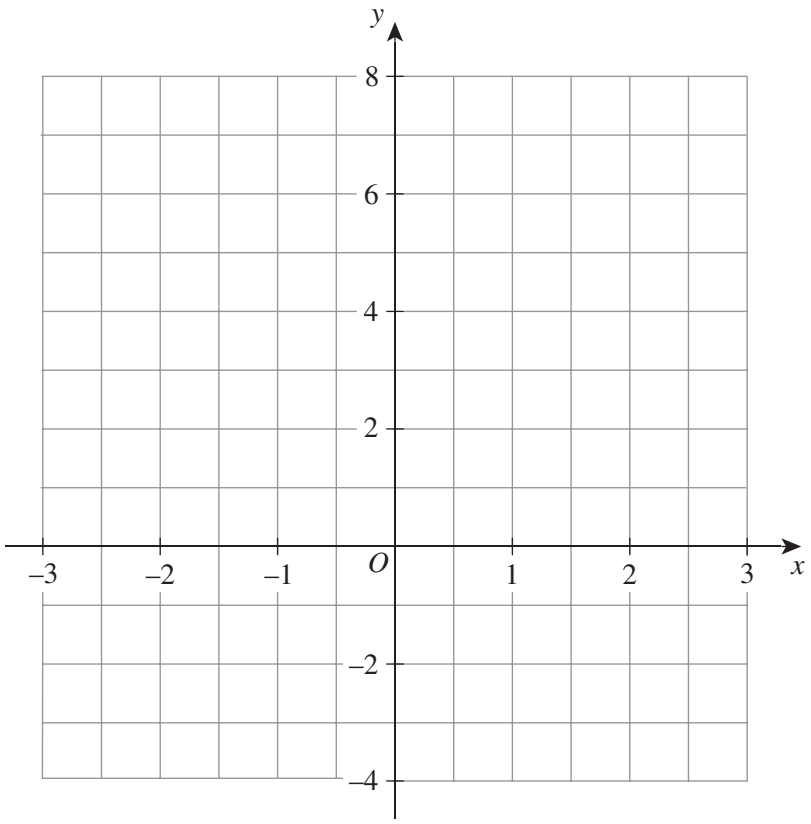
If $f(x)=\begin{cases}x^3-x^2, & x\le 2\\ 4x-1, & x>2\end{cases}$, calculate $f(2)-f(-2)$. **2**

Question 15 (3 marks)

Solve the equations $x-y=2023$ and $x^2-y^2=2023$ simultaneously. **3**

Question 16 (6 marks)

(a) Sketch the graph of $f(x) = |2x + 2|$ in the domain of $(-2, 2]$. Label the axes intercepts and endpoints with their coordinates. **3**



Question 16 continues on page 11

Question 16 (continued)

(b) $f(x) = |2x + 2|$ is reflected in the y-axis. **1**

State the coordinates of the point that is NOT affected by this transformation.

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(c) $(a, f(a))$ is a point on $f(x) = |2x + 2|$. After the graph is reflected in both axes, the resulting image is $\left(\frac{3}{2}, -f(a)\right)$. **2**

Find $f(a)$.

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End of Question 16

Question 17 (3 marks)

The graphs of $y = \sin(x)$ and $y = m\cos(x)$ have a point of intersection at $x = \frac{\pi}{6}$.

(a) Find the value of m. **2**

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(b) The two graphs have a second point of intersection where $0 \leq x \leq 2\pi$.
Find the x-coordinate of the second point. **1**

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Question 18 (2 marks)

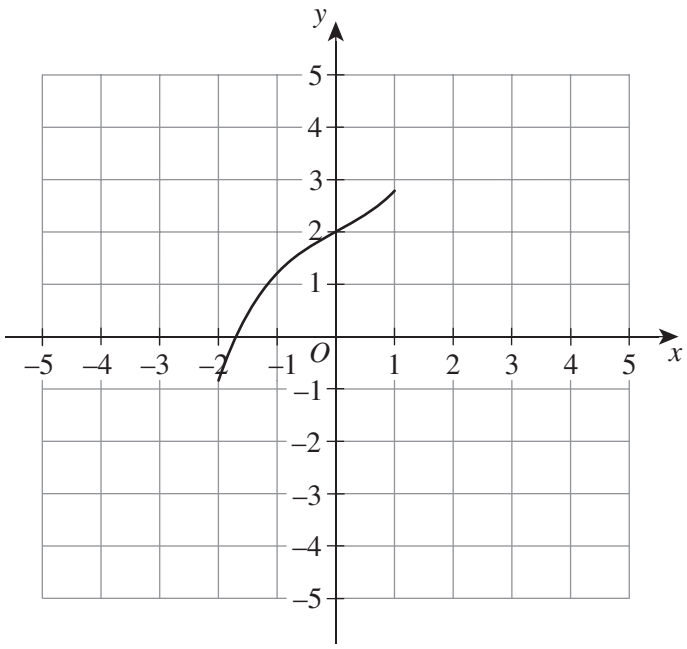
If $\sqrt{2}\tan(\theta)+1=0$ and $90° \le \theta \le 270°$, solve for θ. **2**

Question 19 (2 marks)

Using $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, find the derivative of $f(x)=5x-ax^2$ in terms of a. **2**

Question 20 (3 marks)

The graph shows part of a relation with the domain $-4 \leq x \leq 4$.



(a) Explain why the graph is a one-to-one function with reference to the relevant line test(s). **1**

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(b) Draw an extension of the graph so that it represents a many-to-one function in the interval $-4 \leq x \leq 1$. **1**

(c) Draw an extension of the graph so that it represents a many-to-many relation for the domain $-4 \leq x \leq 4$. **1**

Question 21 (3 marks)

Consider $f(x) = \dfrac{e^{2x}}{x^2 - 2x - 2}$. **3**

Calculate $f'(\sqrt{2})$ in the simplest exact form.

Question 22 (3 marks)

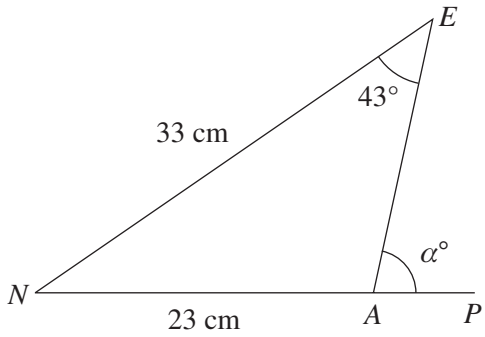
For the random variable x, the following probability distribution is given. **3**

x	–2	–1	0	K
$P(x)$	$\frac{1}{20}$	$P(-1)$	$P(0)$	$P(K)$

If the probabilities are calculated using the equation $P(x) = \frac{2x+5}{20}$, find K.

Question 23 (3 marks)

The diagram shows a triangle where $NE = 33$ cm, $NA = 23$ cm, $\angle NEA = 43°$ and $\angle EAP = \alpha°$ is acute. **3**



NOT TO SCALE

Find the size of $\angle NAE$, correct to the nearest degree.

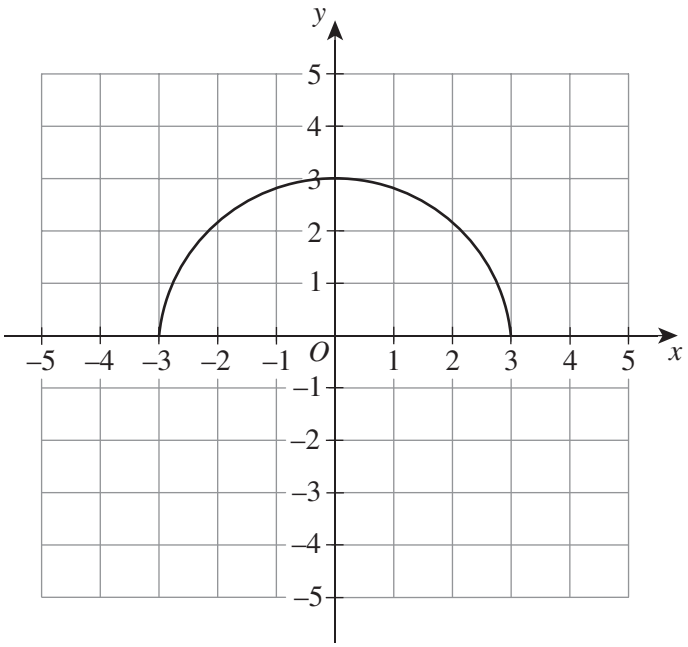
Question 24 (3 marks)

Eflal is approximately 170 cm tall. To calculate the height of a building, she stands at a point and measures the angle of elevation to the top of the building as 45°. She then walks 25.7 metres further away from the building and measures the angle of elevation as 30°. **3**

Find the height of the building, correct to the nearest metre. Draw a diagram to support your response.

Question 25 (6 marks)

The graph shows a semicircle with the equation $y = \sqrt{9 - x^2}$.



(a) State the range of the semicircle. **1**

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(b) (i) On the number plane above, sketch the parabola $y = x^2 - 4$ and label all intercepts. **1**

Question 25 continues on page 20

Question 25 (continued)

(ii) Solve $y = x^2 - 4$ and $y = \sqrt{9 - x^2}$ simultaneously to find the x-coordinates of the intersection points of the graphs. Give your answers correct to two decimal places. **4**

End of Question 25

Question 26 (4 marks)

The graph of $f(x) = \sqrt{x} - 1$ has a normal with the equation $y = -6x + b$, where b is a constant. **4**

Find b.

Question 27 (4 marks)

An electric car company assembles their cars in factories in the USA and Germany. Let a be the proportion of cars that are assembled in the USA. The rest of the cars are assembled in Germany.

Assume that 60% of the cars assembled in the USA are white and 20% of the cars assembled in Germany are white.

(a) A car is selected at random. **1**

In terms of a, what is the probability that the car is white?

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(b) A family wins one of the company's cars in a competition. The car is selected at random.

(i) Given that the winning car is white, calculate the probability that it was assembled in the USA. Give your answer in terms of a. **2**

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(ii) If the probability that this car was assembled in the USA is 0.9, find the value of a. **1**

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Question 28 (4 marks)

The number of stories, S, that Derya shares on her social media account on any given day is a random variable with a probability distribution as shown.

S	0	1	2	3
$P(S = s)$	a	0.2	0.5	b

On average, Derya shares 1.8 stories per day.

(a) Show that $P(S = 0) = a = 0.1$. **2**

Question 28 continues on page 24

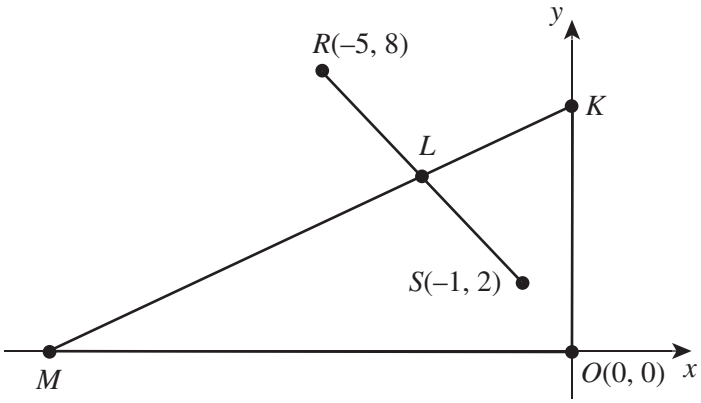
Question 28 (continued)

(b) Last weekend, Derya shared stories on her social media platform on both Saturday and Sunday. What is the probability that she shared a total of three stories over the weekend? **2**

End of Question 28

Question 29 (5 marks)

The diagram shows ΔKOM, where L is the midpoint of $R(-5, 8)$ and $S(-1, 2)$, and lines RS and KM are perpendicular to each other. **5**



Calculate the area of ΔKOM.

Question 30 (3 marks)

Prove that $\tan(180° + \theta°)\left(2 + \cot^2(\theta°)\right) + \tan(90° - \theta°) = 2\sec(\theta°)\text{cosec}(\theta°)$.

Question 31 (5 marks)

Bitcoin is a cryptocurrency. New Bitcoins are produced through cryptocurrency mining. When Bitcoin was introduced as a cryptocurrency at the beginning of 2009, there were 21 000 000 Bitcoins available to be mined. **5**

Bitcoin mining can be modelled using the function

$$B(t) = 21 - a \times e^{-kt}$$

where B is the number of Bitcoins mined, in millions, and t is the number of years. After 12 years, at the beginning of 2021, 80% of the Bitcoins had been mined.

Calculate the rate of change of Bitcoin mining at the end of 2023. Give your answer correct to the nearest hundred thousand per year.

End of paper

Section II extra writing space

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MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

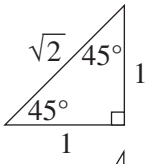
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}},\ \cos A = \frac{\text{adj}}{\text{hyp}},\ \tan A = \frac{\text{opp}}{\text{adj}}$$

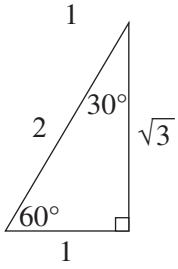
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\text{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

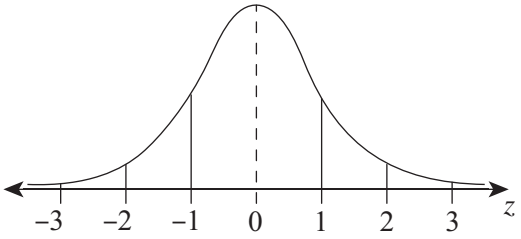
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 - 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)dx$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,\ 1,\ \ldots,\ n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 - \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$
where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$
where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$
and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r\left(\cos\theta + i\sin\theta\right)$$
$$= re^{i\theta}$$

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Final Examination 2023

Neap NSW Year 11 Mathematics Advanced

DIRECTIONS:

Write your name in the space provided.
Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.
Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one** oval per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: _____

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I
MULTIPLE-CHOICE ANSWER SHEET

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II