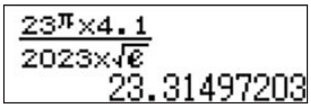
Neap

Final Examination 2023

NSW Year 11 Mathematics Advanced

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **A** Using an approved calculator to evaluate the expression gives: 	MA–F1 Working with Functions MA11–8 Band 2
Question 2 **B** Using the addition rule gives: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.5 = a + 2a - 0.1$ $a = 0.2$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 2–3
Question 3 **C** **C** is correct. This graph represents an odd function as there is point symmetry about the origin. **A** is incorrect. This graph represents neither an odd nor even function. **B** and **D** are incorrect. These graphs represent even functions as they are symmetrical about the *y*-axis.	MA–F1 Working with Functions MA11–1 Bands 2–3
Question 4 **D** Using the change of base law gives: $\log_a x = \dfrac{\log_b x}{\log_b a}$ $\dfrac{\ln 7}{\ln 4} = \dfrac{\log_e 7}{\log_e 4}$ $= \log_4 7$	MA–E1 Logarithms and Exponentials MA11–6 Bands 2–3
Question 5 **B** The visible letters depend on which letter is not visible because it lies against the table. Each letter has a chance of $\frac{1}{4}$ to lie against the table. For the letters E, F and H, the letter G must lie against the table. For the letters E, G and H, the letter F must lie against the table. $P(\text{G}) + P(\text{F}) = \frac{1}{4} + \frac{1}{4}$ $= \frac{1}{2}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4

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Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 6 **A** For two x-intercepts, the discriminant must be positive. Therefore: $\Delta = 7^2 - 4\times -5\times -a > 0$ $49 - 20a > 0$ $a < \frac{49}{20}$ $a < 2.45$ The value of a must be less than 2.45. Checking option **A** gives: $\sin(\pi) = 0 < 2.45$ Checking option **B** gives: $2.5 > 2.45$ Checking option **C** gives: $\pi \approx 3.14 > 2.45$ Checking option **D** gives: $\ln(e^{23}) = 23 > 2.45$ Therefore, only option **A** is less than 2.45.	MA–F1 Working with Functions MA11–1 Bands 4–5
Question 7 **A** Finding the value of k by substituting $P = 5$ and $D = 28$ gives: $P = \frac{k}{D}$ $k = P \times D$ $k = 5 \times 28$ $= 140$ Finding the number of days after the number of people in the household increased gives: $P = \frac{140}{D}$ $5 + 2 = \frac{140}{D_{new}}$ $D_{new} = 20$ Therefore, the toilet paper lasted 28 – 20 = 8 days fewer.	MA–F1 Working with Functions MA11–2 Bands 3–4

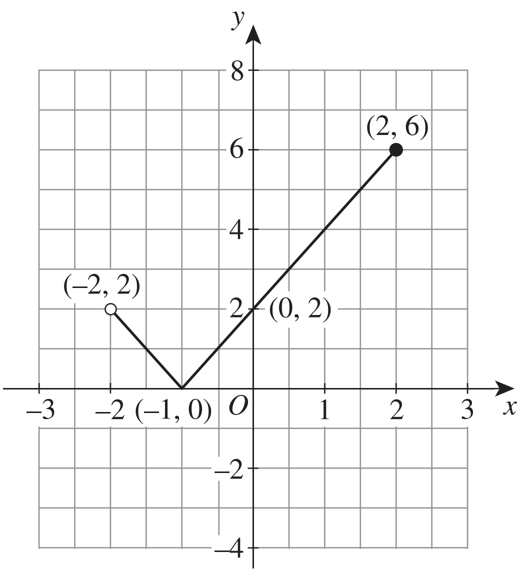
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 8 **C** **C** is correct. $\text{area} = \dfrac{a \times b \times \sin C}{2}$ $42 = \dfrac{x^2 \times \sin(42°)}{2}$ Due to complementary angles: $\sin(42°) = \cos(48°)$ $\dfrac{x^2 \times \cos(48°)}{2} = 42$ **A** and **B** are incorrect. 42 is not a side length. **D** is incorrect. This option is missing the square root shown below. $x = \sqrt{\dfrac{84}{\sin\left(\dfrac{42\pi}{180}\right)}}$	MA–T1 Trigonometry and Measure of Angles MA11–3 Bands 4–6
Question 9 **A** *A* and *B* are parabolas with maximum and minimum turning points, respectively. Finding the maximum value of *A* gives: $a = \dfrac{-8}{2 \times -1}$ $= 4$ $A_{\max} = -\left(4^2\right) + 8 \times 4 + 1$ $= 17$ Finding the minimum value of *B* gives: $b = \dfrac{-18}{2 \times 1}$ $= -9$ $B_{\min} = (-9)^2 + 18 \times -9 + 5$ $= -76$ Therefore, the sum of the maximum value of *A* and minimum value of *B* is: $A_{\max} + B_{\min} = 17 + -76$ $= -59$	MA–F1 Working with Functions MA11–1 Bands 4–5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 10 **B** Applying the chain rule gives: $\dfrac{df\left(x^3\right)}{dx} = \left(x^3\right)' \times f'\left(x^3\right)$ $= 3x^2 \times f'\left(x^3\right)$ $\dfrac{dg(x+1)}{dx} = (x+1)' \times g'(x+1)$ $= 1 \times g'(x+1)$ $= g'(x+1)$ Applying the product rule gives: $\dfrac{df\left(x^3\right)}{dx} \times g(x+1) + f\left(x^3\right) \times \dfrac{dg(x+1)}{dx}$ $= 3x^2 \times f'\left(x^3\right) \times g(x+1) + f\left(x^3\right) \times g'(x+1)$ $= 3x^2 f'\left(x^3\right) g(x+1) + f\left(x^3\right) g'(x+1)$	MA–C1 Introduction to Differentiation MA11–5 Bands 5–6

SECTION II

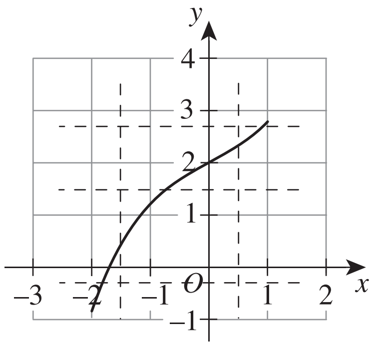
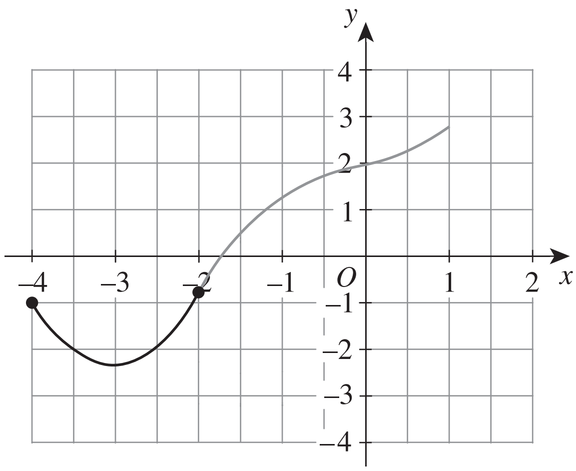
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
$18x = 9x^2$ $18x - 9x^2 = 0$ $9x(2-x) = 0$ $x = 0,\ x = 2$	MA–F1 Working with Functions MA11–1 Bands 2–3 • Provides the correct solution . . . . . . 2 • Makes progress towards factorising the equation OR equivalent merit. 1
Question 12	
$\left(3\sqrt{5}-\sqrt{3}\right)\left(5\sqrt{3}+\sqrt{5}\right) = 15\sqrt{15}+15-15-\sqrt{15}$ $= 14\sqrt{15}$	MA–F1 Working with Functions MA11–1 Bands 2–3 • Provides the correct solution . . . . . . 2 • Expands the product 1
Question 13	
$\left\|\frac{6-5x}{2}\right\| = 3$ $\frac{6-5x}{2} = 3$ $6-5x = 6$ $-5x = 0$ $x = 0$ $\frac{6-5x}{2} = -3$ $6-5x = -6$ $-5x = -12$ $x = 2.4$ $\therefore x = 0, 2.4$	MA–F1 Working with Functions MA11–1 Bands 3–4 • Provides the correct solution . . . . . . 2 • Attempts to deal with the absolute value OR equivalent merit. 1

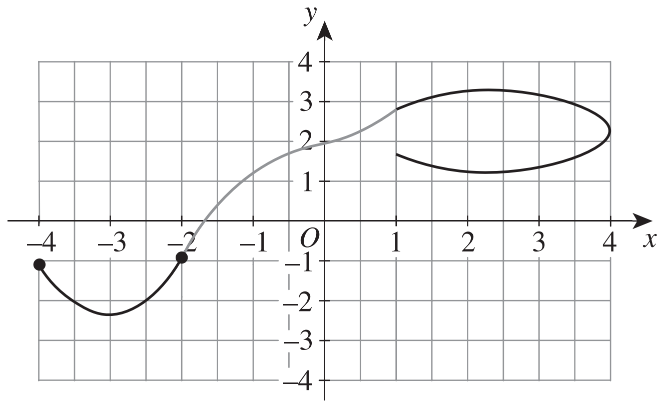
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
$f(x)=\begin{cases} x^3-x^2, & x\le 2 \\ 4x-1, & x>2 \end{cases}$ $f(2)=2^3-2^2$ $=4$ $f(-2)=(-2)^3-(-2)^2$ $=-12$ $\therefore f(2)-f(-2)=4--12=16$	MA–F1 Working with Functions MA11–1 Bands 3–4 • Provides the correct solution . . . . . . 2 • Attempts to substitute into the correct rule OR equivalent merit. 1
Question 15	
$x-y=2023 \quad (1)$ $x^2-y^2=2023 \quad (2)$ Using the difference of two squares rule for (2) gives: $(x-y)(x+y)=2023 \quad (3)$ Substituting (1) into (3) gives: $2023(x+y)=2023$ $x+y=1 \quad (4)$ Solving (1) and (4) simultaneously gives: $2x=2024$ $x=1012$ Substituting $x = 1012$ into (1) gives: $1012-y=2023$ $y=-1011$ $\therefore x=1012,\ y=-1011$	MA–F1 Working with Functions MA11–1 Bands 3–4 • Provides the correct solution . . . . . . 3 • Uses the difference of two squares rule OR substitutes $x = 2023 + y$. AND • Makes substantial progress towards cancelling one of the variables. . . . . 2 • Uses the difference of two squares rule OR substitutes $x = 2023 + y$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 16	
(a) Finding the coordinates of the endpoints: $f(-2) = \|2 \times -2 + 2\|$ $= 2$ The first endpoint is (–2, 2). $f(2) = \|2 \times 2 + 2\|$ $= 6$ The second endpoint is (2, 6). Finding the coordinates of the intercepts: $f(0) = \|2 \times 0 + 2\|$ $= 2$ The y-intercept is (0, 2). $f(x) = 0$ $0 = \|2x + 2\|$ $x = -1$ The x-intercept is (–1, 0).  	MA–F1 Working with Functions MA11–1 Bands 3–4 • Finds the coordinates of the endpoints. AND • Finds the coordinates of the intercepts. AND • Sketches a graph that has the correct shape AND shows the correct endpoints. . . . . . . 3 • Any TWO of the above points. . . . . 2 • Any ONE of the above points 1
(b) (0, 2) *Note: The y-intercept is not affected by a reflection in the y-axis.*	MA–F1 Working with Functions MA11–1 Bands 2–3 • Provides the correct solution 1

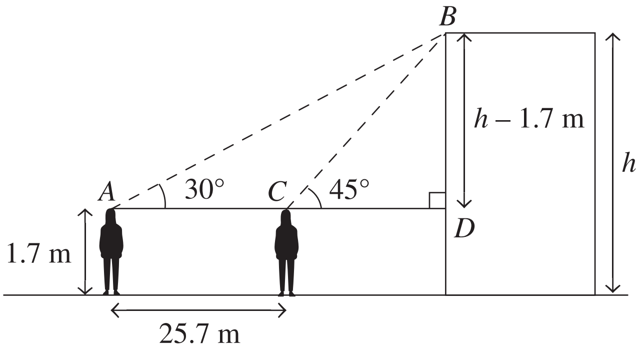
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) When reflected in the y-axis, a becomes $-a$. Since $-a=\frac{3}{2}$ after the y-axis reflection, then $a=-\frac{3}{2}$. Substituting $a=-\frac{3}{2}$ into $f(x)$ gives: $f\left(-\frac{3}{2}\right)=\left\|2\times-\frac{3}{2}+2\right\|$ $=\|-1\|$ $=1$ Therefore, $f(a)=1$.	MA–F1 Working with Functions MA11–1 Bands 3–5 • Provides the correct solution . . . . . . 2 • Finds $a=-\frac{3}{2}$ 1
Question 17	
(a) Equating the RHS in both equations and substituting $x=\frac{\pi}{6}$ gives: $m\cos\frac{\pi}{6}=\sin\frac{\pi}{6}$ $m\times\frac{\sqrt{3}}{2}=\frac{1}{2}$ $m=\frac{1}{\sqrt{3}}$ $=\frac{\sqrt{3}}{3}$	MA–T1 Trigonometry and Measure of Angles MA11–1, 11–3 Bands 3–4 • Provides the correct solution . . . . . . 2 • Provides the exact evaluation of trigonometric ratios for $\frac{\pi}{6}$ 1
(b) $\frac{\sqrt{3}}{3}\cos(x)=\sin(x)$ $\frac{\sqrt{3}}{3}=\frac{\sin(x)}{\cos(x)}$ $\tan(x)=\frac{\sqrt{3}}{3}$ $x=\frac{\pi}{6},\ \pi+\frac{\pi}{6}$ $=\frac{\pi}{6},\ \frac{7\pi}{6}$ As $\frac{\pi}{6}$ is the x-coordinate for the first point of intersection, the x-coordinate of the second point of intersection must be $\frac{7\pi}{6}$. *Note: Consequential on answer to **Question 17(a)**.*	MA–T1 Trigonometry and Measure of Angles MA11–1, 11–3 Bands 3–4 • Provides the correct solution 1

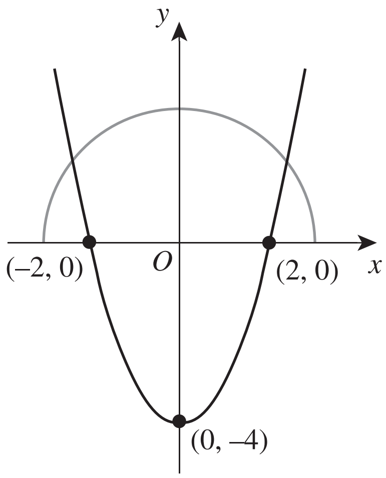
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 18	
$\sqrt{2}\tan\theta + 1 = 0$ $\theta = \tan^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ $= -35.26°$ Since $90 \ \le \theta \le 270°$: $\theta = -35.26° + 180°$ $= 144.74°$	MA–T1 Trigonometry and Measure of Angles MA11–1, 11–3, 11–9 Bands 4–5 • Provides the correct solution . . . . . . 2 • Attempts to use tangent inverse OR equivalent merit 1
Question 19	
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{5(x+h) - a(x+h)^2 - (5x - ax^2)}{h}$ $= \lim_{h \to 0} \frac{5x + 5h - a\left(x^2 + 2xh + h^2\right) - 5x + ax^2}{h}$ $= \lim_{h \to 0} \frac{5x + 5h - ax^2 - 2axh - ah^2 - 5x + ax^2}{h}$ $= \lim_{h \to 0} \frac{5h - 2axh - ah^2}{h}$ $= \lim_{h \to 0} \frac{\not{h}(5 - 2ax - ah)}{\not{h}}$ $= 5 - 2ax - a \times 0$ $= 5 - 2ax$	MA–C1 Introduction to Differentiation MA11–5 Bands 4–5 • Provides the correct solution . . . . . . 2 • Makes substantial progress towards cancelling the factor h 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 20	
(a) The graph passes both vertical and horizontal line tests, so it is a one-to-one function. 	MA–F1 Working with Functions MA11–1 Bands 2–3 • Provides the correct solution 1
(b)  *Note: The student's graph may vary, but it must fail the horizontal line test and must not continue past x = –4. Parabolic graphs are acceptable.*	MA–F1 Working with Functions MA11–1 Bands 3–4 • Provides the correct solution 1

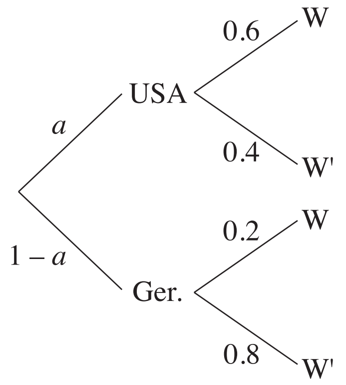
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c)  *Note: The student's graph may vary, but it must fail both the horizontal and vertical line tests and must not continue past x = 1 and x = 4. Circular graphs are acceptable.*	MA–F1 Working with Functions MA11–1 Bands 3–4 • Provides the correct solution 1
Question 21	
$f'(x)=\dfrac{2e^{2x}\left(x^2-2x-2\right)-(2x-2)e^{2x}}{\left(x^2-2x-2\right)^2}$ $=\dfrac{2e^{2x}\left(x^2-2x-2\right)-2e^{2x}(x-1)}{\left(x^2-2x-2\right)^2}$ $=\dfrac{2e^{2x}\left(x^2-3x-1\right)}{\left(x^2-2x-2\right)^2}$ $f'\left(\sqrt{2}\right)=\dfrac{2e^{2\sqrt{2}}\left(\left(\sqrt{2}\right)^2-3\sqrt{2}-1\right)}{\left(\left(\sqrt{2}\right)^2-2\sqrt{2}-2\right)^2}$ $=\dfrac{2e^{2\sqrt{2}}\left(1-3\sqrt{2}\right)}{8}$ $=\dfrac{e^{2\sqrt{2}}\left(1-3\sqrt{2}\right)}{4}$	MA–C1 Introduction to Differentiation MA–E1 Logarithms and Exponentials MA11–5, 11–6 Bands 3–4 • Provides the correct solution . . . . . . 3 • Applies the quotient rule. OR • Attempts to apply the quotient rule AND simplifies the equation after substituting $\sqrt{2}$ . . . . 2 • Attempts to apply the quotient rule OR simplifies the equation after substituting $\sqrt{2}$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 22	
Finding $P(-1)$ gives: $P(-1)=\frac{2\times -1+5}{20}$ $=\frac{3}{20}$ Finding $P(0)$ gives: $P(0)=\frac{2\times 0+5}{20}$ $=\frac{5}{20}$ Finding $P(K)$ gives: $P(K)=\frac{2K+5}{20}$ As all probabilities in the distribution add up to 1, equating all probabilities to 1 to find K gives: $P(-2)+P(-1)+P(0)+P(K)=1$ $\frac{1}{20}+\frac{3}{20}+\frac{5}{20}+\frac{2K+5}{20}=1$ $K=3$	MA–S1 Probability and Discrete Probability Distributions MA11–1, 11–7 Bands 2–4 • Provides the correct solution . . . . . . 3 • Finds $P(-1)$, $P(0)$ and $P(K)$ by substituting into $P(x)$. AND • Applies $\sum P(x)=1$ . . . . 2 • Any ONE of the above points 1
Question 23	
Let $\angle NAE=\theta$. $\frac{\sin\theta}{33}=\frac{\sin(43°)}{23}$ $\theta=\sin^{-1}\left(\frac{33\sin(43°)}{23}\right)$ $\approx 78°$ Since α is an acute angle and $\theta+\alpha=180°$, θ must be an obtuse angle. Using the ambiguous case gives: $\angle NAE=\theta$ $\approx 180°-78°$ $\approx 102°$	MA–T1 Trigonometry and Measure of Angles MA11–3, 11–9 Bands 4–5 • Provides the correct solution . . . . . . 3 • Applies the sine rule. AND • Finds $\theta=78°$ . . . . 2 • Applies the sine rule. OR • Identifies the ambiguous case 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 24	
 Let the height of the building be h. Given that ΔBCD is isosceles: $CD = BD = h - 1.7$ $\therefore AD = 25.7 + h - 1.7$ $= 24 + h$ Using trigonometric ratios to find h gives: $\tan(30°) = \frac{BD}{AD}$ $\frac{\sqrt{3}}{3} = \frac{h-1.7}{24+h}$ $24\sqrt{3} + h\sqrt{3} = 3h - 5.1$ $24\sqrt{3} + 5.1 = 3h - h\sqrt{3}$ $24\sqrt{3} + 5.1 = h(3 - \sqrt{3})$ $h = \frac{24\sqrt{3}+5.1}{3-\sqrt{3}}$ $= 36.8069$ m ≈ 37 m	MA–T1 Trigonometry and Measure of Angles MA11–1, 11–3 Bands 4–5 • Provides the correct solution . . . . . . 3 • Draws a diagram. AND • Uses trigonometric ratios to make significant progress towards finding the height . . . . 2 • Draws a diagram OR • Identifies that tan(30°) should be used with ΔABD OR equivalent merit. 1
Question 25	
(a) $0 \le y \le 3$ **OR** [0, 3]	MA–F1 Working with Functions MA11–1 Bands 2–3 • Provides the correct solution 1

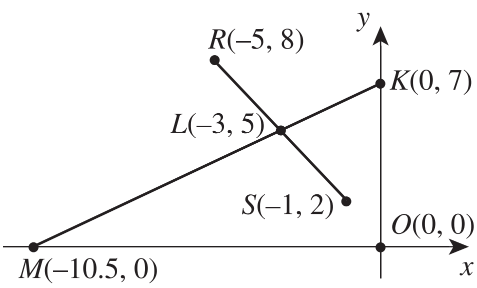
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) (i) 	MA–F1 Working with Functions MA11–1 Bands 3–4 • Sketches the correct graph with all THREE intercepts labelled 1
(ii) $y = x^2 - 4$ (1) $y = \sqrt{9 - x^2}$ (2) Equating (1) and (2) gives: $x^2 - 4 = \sqrt{9 - x^2}$ Let x^2 be A. Therefore: $A - 4 = \sqrt{9 - A}$ $(A - 4)^2 = \left(\sqrt{9 - A}\right)^2$ $A^2 - 8A + 16 = 9 - A$ $A^2 - 7A + 7 = 0$ Using the quadratic formula gives: $A = \dfrac{-(-7) \pm \sqrt{-7^2 - 4 \times 1 \times 7}}{2 \times 1}$ $= \dfrac{7 \pm \sqrt{21}}{2}$ $\approx 1.2087,\ 5.7913$ As $A = x^2$: $x \approx \pm\sqrt{1.2087}$ or $\pm\sqrt{5.7913}$ $\approx \pm 1.099$ or ± 2.407 As shown by the graph drawn in part (i), the intersection points with the upper semicircle occur when $x \approx \pm 2.407$. $\therefore x \approx \pm 2.41$ *Note: Consequential on answer to* ***Question 25(b)(i)***.	MA–F1 Working with Functions MA11–1, 11–8, 11–9 Bands 4–6 • Provides the correct solution . . . . . . 4 • Makes substantial progress towards calculating the values of A . . . . 3 • Makes progress towards using the quadratic formula OR equivalent merit. . . . . 2 • Attempts to take the square of both sides OR substitute $x^2 = A$. OR • Provides some relevant working . . . 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 26	
The gradient of the normal is –6. Finding the gradient of the tangent gives: $m_N \times m_T = -1$ $-6 \times m_T = -1$ $m_T = \frac{-1}{-6}$ $= \frac{1}{6}$ Finding the derivative of the function gives: $f'(x) = \frac{1}{2\sqrt{x}}$ Thus, equating the derivative and the gradient of the tangent gives: $f'(x) = m_T$ $\frac{1}{2\sqrt{x}} = \frac{1}{6}$ $\therefore 2\sqrt{x} = 6$ $x = 9$ Substituting $x = 9$ into the function gives: $f(9) = \sqrt{9} - 1$ $= 2$ Therefore, the normal passes through the point (9, 2) and the equation of the normal is: $y_N - 2 = -6(x - 9)$ $y_N = -6x + 54 + 2$ $= -6x + 56$ $\therefore b = 56$	MA–C1 Introduction to Differentiation MA–F1 Working with Functions MA11–1, 11–5 Bands 3–5 • Finds the gradient of the tangent. AND • Equates the derivative of the function to the gradient of the tangent. AND • Finds the coordinates (9, 2). AND • Uses the completed equation of the normal to find the value of b . . . 4 • Any THREE of the above points. . . 3 • Any TWO of the above points. . . . . 2 • Any ONE of the above points 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 27	
(a)  $P(\text{white}) = P(\text{USA} \cap \text{white}) + P(\text{Germany} \cap \text{white})$ $= a \times 0.6 + (1-a) \times 0.2$ $= 0.6a + 0.2 - 0.2a$ $= 0.4a + 0.2$ *Note: Diagrams are not required to achieve full marks, but may be provided to develop the response.*	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4 • Provides the correct solution 1
(b) (i) $P(\text{USA} \mid \text{white}) = \dfrac{P(\text{USA} \cap \text{white})}{P(\text{white})}$ $= \dfrac{0.6a}{0.4a + 0.2}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–5 • Provides the correct solution . . . . . . 2 • Makes progress towards calculating the conditional probability OR equivalent merit . . . 1
(ii) $\dfrac{0.6a}{0.4a + 0.2} = 0.9$ $0.6a = 0.36a + 0.18$ $0.24a = 0.18$ $a = 0.75$ *Note: Consequential on answer to* ***Question 27(b)(i)****.*	MA–S1 Probability and Discrete Probability Distributions MA11–1, 11–7 Bands 2–3 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
(a) Creating simultaneous equations gives: $\sum_{0}^{3} s \times P(S = s) = 1.8 \quad (1)$ $\sum_{0}^{3} P(S = s) = 1 \quad (2)$ Substituting the values from the probability distribution into (1) gives: $0 \times a + 1 \times 0.2 + 2 \times 0.5 + 3 \times b = 1.8$ $0.2 + 1 + 3b = 1.8$ $b = 0.2$ Substituting the values from the probability distribution into (2) gives: $a + 0.2 + 0.5 + b = 1 \quad (3)$ Substituting $b = 0.2$ into (3) gives: $a + 0.2 + 0.5 + 0.2 = 1$ $a = 0.1$ $\therefore P(S = 0) = a = 0.1$ (as required)	MA–S1 Probability and Discrete Probability Distributions MA–F1 Working with Functions MA11–1, 11–7 Bands 3–4 • Provides the correct solution . . . . . . 2 • Makes progress towards creating the average value equation OR equivalent merit 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b)	A total of three stories over the weekend requires Derya posting two stories on Saturday and one story on Sunday or one story on Saturday and two stories on Sunday. (Three stories on one day is not possible, as Derya shared stories on both days.) Therefore: $P(S_{\text{Sat}} = 2 \cap S_{\text{Sun}} = 1) = P(S_{\text{Sat}} = 2) \times P(S_{\text{Sun}} = 1)$ $= 0.5 \times 0.2$ $= 0.10$ $P(S_{\text{Sat}} = 1 \cap S_{\text{Sun}} = 2) = P(S_{\text{Sat}} = 1) \times P(S_{\text{Sun}} = 2)$ $= 0.2 \times 0.5$ $= 0.10$ $P(S_{\text{total}} = 3) = 0.10 + 0.10$ $= 0.20$ It is known that Derya shared stories on both days; therefore, $S_{\text{Sat}} \geq 1$ and $S_{\text{Sun}} \geq 1$. $P(S_{\text{Sat}} \geq 1) = P(S_{\text{Sun}} \geq 1)$ $= 0.9$ Hence: $P(S_{\text{Sat}} \geq 1 \cap S_{\text{Sun}} \geq 1) = 0.9 \times 0.9$ $= 0.81$ (This is also reduced sample space probability.) Therefore, $P(S_{\text{total}} = 3 \mid \text{shared stories on both days})$ is: $P(S_{\text{total}} = 3 \mid (S_{\text{Sat}} \geq 1 \text{ and } S_{\text{Sun}} \geq 1))$ $= \dfrac{P(S_{\text{total}} = 3 \cap (S_{\text{Sat}} \geq 1 \text{ and } S_{\text{Sun}} \geq 1))}{P(S_{\text{Sat}} \geq 1 \text{ and } S_{\text{Sun}} \geq 1)}$ $= \dfrac{P(S_{\text{total}} = 3)}{P(S_{\text{Sat}} \geq 1 \cap S_{\text{Sun}} \geq 1)}$ $= \dfrac{0.20}{0.81}$ $= \dfrac{20}{81}$	MA–S1 Probability and Discrete Probability Distributions MA11–7, 11–9 Bands 5–6 • Provides the correct solution . . . . . . 2 • Makes progress towards calculating the reduced sample space. OR • Calculates the probability of Derya posting three stories over successive days. OR • Shows understanding of the conditional probability case OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 29	
Finding the midpoint of R and S gives: $L\left(\frac{-5+-1}{2}, \frac{8+2}{2}\right) = L(-3, 5)$ Finding the gradient of line RS gives: $m_{RS} = \frac{8-2}{-5--1}$ $= -\frac{3}{2}$ Finding the gradient of line KM as it is perpendicular to line RS gives: $m_{KM} \times m_{RS} = -1$ $m_{KM} \times -\frac{3}{2} = -1$ $m_{KM} = \frac{2}{3}$ Finding the equation of line KM gives: $y_{KM} - 5 = \frac{2}{3}(x - -3)$ $y_{KM} = \frac{2}{3}x + 7$ Finding the intercepts of line KM gives: $0 = \frac{2}{3}x + 7$ $x = -10.5$ Therefore, the coordinates of point M are (–10.5, 0). $y_{KM} = \frac{2}{3} \times 0 + 7$ $= 7$ Therefore, the coordinates of point K are (0, 7).  Finding the area of ΔKOM gives: $A = \frac{10.5 \times 7}{2}$ $= 36.75 \text{ units}^2$ *Note: Diagrams are not required to achieve full marks, but may be provided to develop the response.*	MA–F1 Working with Functions MA11–1, 11–2 Bands 4–5 • Calculates m_{RS}. AND • Calculates m_{KM}. AND • Writes the equation of y_{KM}. AND • Calculates the intercepts of y_{KM}. AND • Calculates the area of ΔKOM . . . . . 5 • Any FOUR of the above points . . . . 4 • Any THREE of the above points . . . 3 • Any TWO of the above points. . . . . 2 • Any ONE of the above points 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 30	
$\tan(180°+\theta°)=\tan(\theta°)$ $\tan(90°-\theta°)=\cot(\theta°)$ $\text{LHS}=\tan(\theta°)\left(2+\cot^2(\theta°)\right)+\cot(\theta°)$ $=2\tan(\theta°)+\tan(\theta°)\times\dfrac{1}{\tan^2(\theta°)}+\dfrac{1}{\tan(\theta°)}$ $=2\tan(\theta°)+\dfrac{2}{\tan(\theta°)}$ $=\dfrac{2\tan^2(\theta°)}{\tan(\theta°)}+\dfrac{2}{\tan(\theta°)}$ $=\dfrac{2\left(\tan^2(\theta°)+1\right)}{\tan(\theta°)}$ $=\dfrac{2\sec^2(\theta°)}{\dfrac{\sin(\theta°)}{\cos(\theta°)}}$ $=\dfrac{2}{\cos^2(\theta°)}\times\dfrac{\cos(\theta°)}{\sin(\theta°)}$ $=\dfrac{2}{\cos(\theta°)}\times\dfrac{1}{\sin(\theta°)}$ $=2\sec(\theta°)\text{cosec}(\theta°)$ $=\text{RHS}$	MA–T2 Trigonometric Functions and Identities MA11–1, 11–4 Bands 4–6 • Provides the correct solution . . . . . . 3 • Converts $\tan(180°+\theta°)$ and $\tan(90°-\theta°)$. AND • Makes progress towards manipulating the trigonometric ratios . . . . 2 • Converts $\tan(180°+\theta°)$ and $\tan(90°-\theta°)$. OR • Makes progress towards manipulating the trigonometric ratios 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 31	
When $t = 0$, $B(0) = 0$. Therefore: $0 = 21 - a \times e^{-k \times 0}$ $0 = 21 - a$ $a = 21$ $B(t) = 21 - 21e^{-kt}$ When $t = 12$, $B(12) = 0.8 \times 21$ (80% of 21 000 000): $21 - 21 \times e^{-k \times 12} = 0.8 \times 21$ $21 \times e^{-12k} = 4.2$ $e^{-12k} = 0.2$ $= 5^{-1}$ $\ln e^{-12k} = \ln 5^{-1}$ $-12k = -\ln 5$ $k = \frac{\ln 5}{12}$ Hence, $B(t) = 21 - 21e^{-\frac{\ln 5}{12}t}$. Finding the derivative of $B(t)$ to find the rate of change gives: $B'(t) = 0 - 21 \times -\frac{\ln 5}{12} e^{-\frac{\ln 5}{12}t}$ $= \frac{7 \ln 5}{4} e^{-\frac{\ln 5}{12}t}$ To find the instantaneous rate of change, the t value at the end of 2023 is the same as the beginning of 2024; that is, $t = 15$. Substituting $t = 15$ into the derivative gives: $B'(15) = \frac{7 \ln 5}{4} e^{-\frac{15 \ln 5}{12}}$ $= 0.3767\ldots$ Converting to millions gives: rate $= 0.3767\ldots \times 1\ 000\ 000$ $= 376\ 703.6$ $\approx 400\ 000$ Bitcoins per year	MA–C1 Introduction to Differentiation MA–E1 Logarithms and Exponentials MA–F1 Working with Functions MA11–1, 11–2, 11–6, 11–8, 11–9 Bands 4–6 • Finds $a = 21$. AND • Calculates $k = \frac{\ln 5}{12}$. AND • Finds $B'(t)$. AND • Calculates $B'(15)$. AND • Expresses $B'(15)$ to the nearest hundred thousand . . . . 5 • Any FOUR of the above points . . . . 4 • Any THREE of the above points . . . 3 • Any TWO of the above points. . . . . 2 • Any ONE of the above points 1