Neap

Trial Examination 2022

HSC Year 12 Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

SECTION I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II – 90 marks (pages 7–33)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2022 HSC Year 12 Mathematics Advanced examination.

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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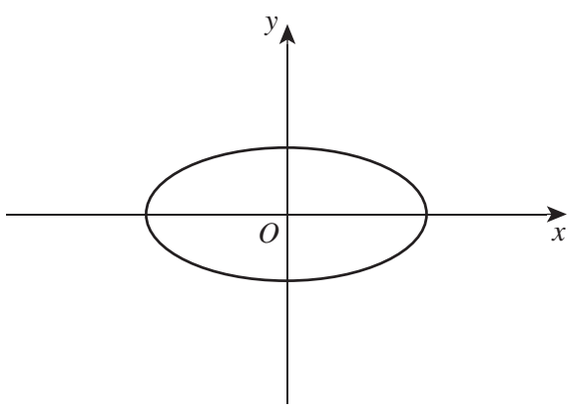
1 The diagram shows a curve.



Which type of relationship best describes the curve?

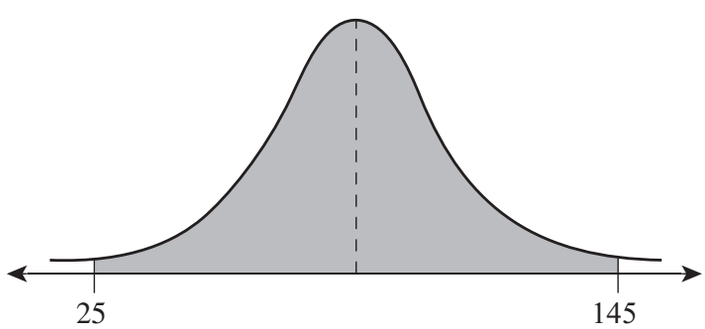
A. one-to-one

B. one-to-many

C. many-to-one

D. many-to-many

2 The graph shows a normal distribution. Approximately 99.7% of the area under the curve is bounded by $x = 25$ and $x = 145$.



What are the values of μ and $\text{Var}(X)$?

A. $\mu = 60$ and $\text{Var}(X) = 20$

B. $\mu = 60$ and $\text{Var}(X) = 400$

C. $\mu = 85$ and $\text{Var}(X) = 20$

D. $\mu = 85$ and $\text{Var}(X) = 400$

3 A circle has the equation $(x + 2)^2 + (y + 3)^2 = d$.
What is the value of d such that the x-axis is a tangent to the circle?

A. 2

B. 3

C. 4

D. 9

4 Phoebe wants to borrow money from the bank to buy a caravan. The table is used to calculate the monthly repayments for the loan.

Monthly repayments

	Loan term length				
Amount borrowed	*10 years*	*15 years*	*20 years*	*25 years*	*30 years*
$100 000	$1213.28	$955.65	$836.44	$771.82	$733.76
$110 000	$1334.60	$1051.22	$920.08	$849.00	$807.14
$120 000	$1455.96	$1146.78	$1003.37	$926.18	$880.52
$130 000	$1577.26	$1242.35	$1087.37	$1003.36	$953.89
$140 000	$1698.59	$1337.91	$1171.02	$1080.54	$1027.27
$150 000	$1819.91	$1433.48	$1254.66	$1157.72	$1100.65
$160 000	$1941.24	$1529.04	$1338.30	$1234.91	$1174.02

Phoebe is considering two options.

- Option 1: Borrow $100 000 for 20 years.
- Option 2: Borrow $110 000 for 30 years.

Phoebe calculates that she will pay more interest by choosing option 1 than choosing option 2.

Calculate the difference in interest between the two options.

A. $7 485.40

B. $79 824.80

C. $200 745.60

D. $290 570.40

5 Consider the two sets $A = \{5, 7, 9\}$ and $B = \{10, 11, 12\}$. A number is randomly chosen from each set.

What is the probability of selecting a number from each set that have a sum that is equal to or greater than 20?

A. $\frac{1}{9}$

B. $\frac{1}{6}$

C. $\frac{2}{9}$

D. $\frac{1}{3}$

6 Given that $f(x) = \int_2^x \frac{1}{1+t^3} dt$, what is the value of $f'(2)$?

A. $-\frac{4}{27}$

B. 0

C. $\frac{1}{9}$

D. ln5

7 The probability distribution of a discrete random variable X is shown.

x	1	2	5	6	8
$P(X = x)$	0.05	0.15	0.4	0.2	0.2

If μ is the mean of X, then $P(X > \mu)$ is

A. 0.4

B. 0.8

C. 5.15

D. 19

8 Consider the graphs of $y = px^2$ and $y = qx^2 + r$.

The graphs will have no points of intersection for the real constants p, q and r provided that

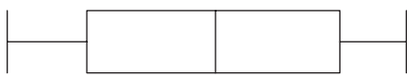
A. $r > 0$ and $p < q$.

B. $r > 0$ and $p > q$.

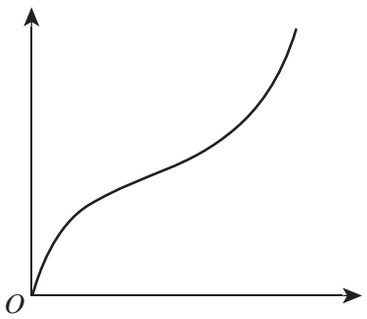
C. $p < 0$ and $q > 0$ for all real values of r.

D. $p > 0$ and $q < 0$ for all real values of r.

9 The box-plot and cumulative frequency diagram for a dataset are shown.



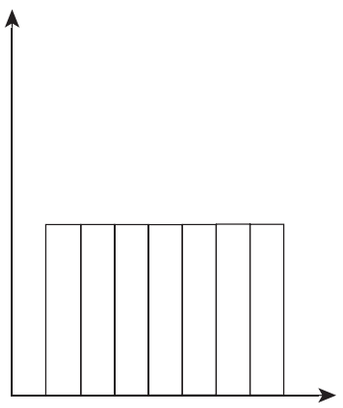
Box-plot



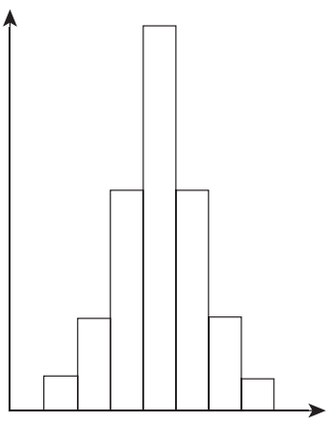
Cumulative frequency diagram

Which of the following frequency histograms best represents the dataset?

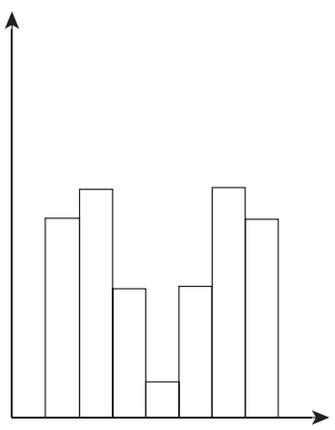
A.



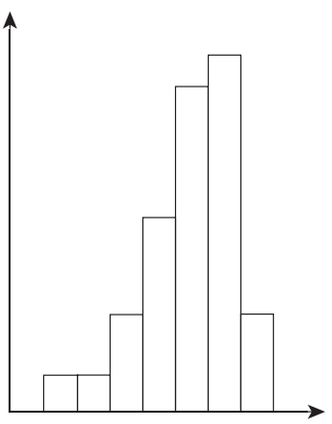
B.



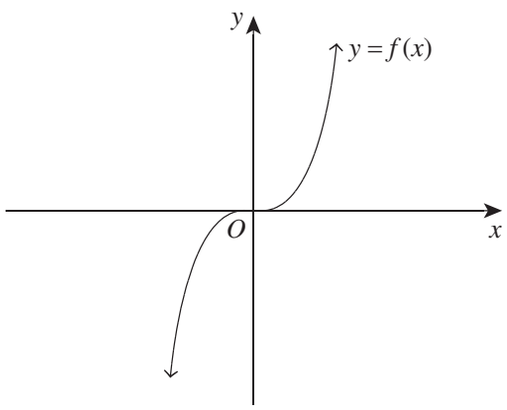
C.

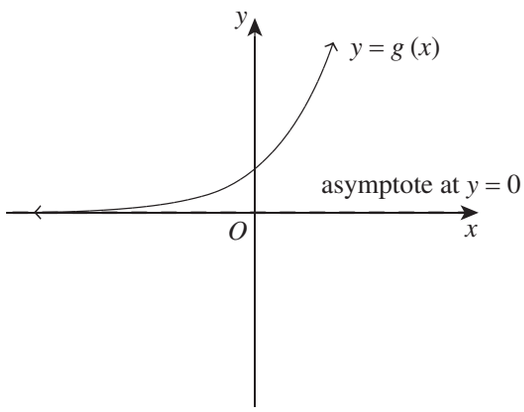


D.



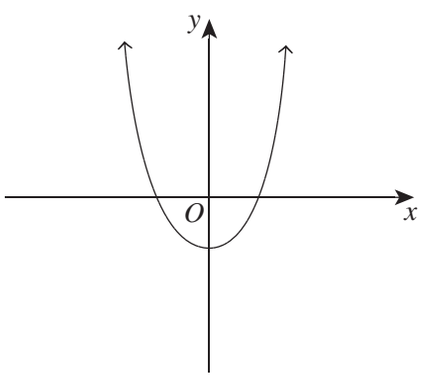
10 The graphs of $f(x)$ and $g(x)$ are shown.



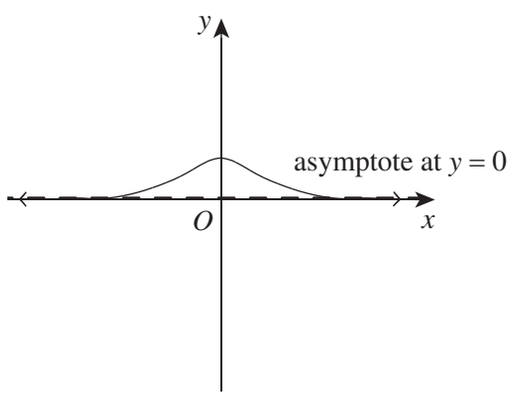


Which of the following best represents the graph of $y = f\left(g(-x)\right)$?

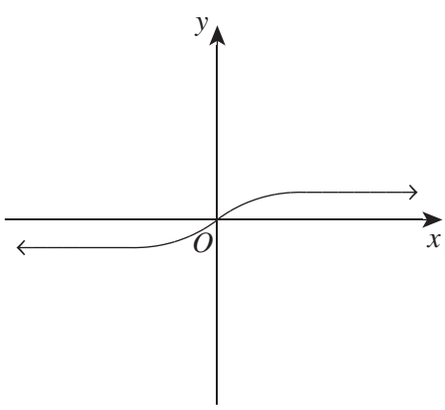
A.



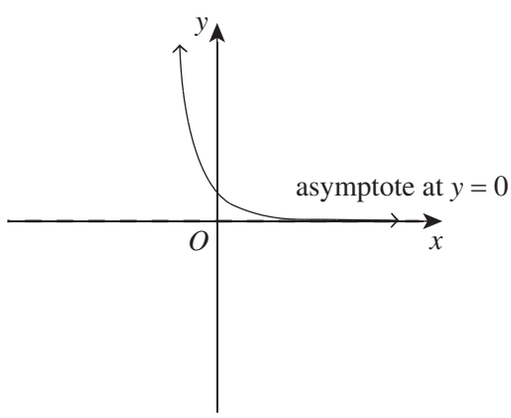
B.



C.



D.



HSC Year 12 Mathematics Advanced

Section II Answer Booklet 1

SECTION II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–21 (36 marks)
Booklet 2 — Attempt Questions 22–31 (54 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 18 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)
Solve $|2x-3|=4$. **2**

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Question 12 (2 marks)
The function $y=f(x)$ is defined as **2**

$$f(x)=\begin{cases} 1 & \text{for} \quad -2\le x\le 0 \\ -x^2+1 & \text{for} \quad x\ge 0 \end{cases}$$

By sketching $y=f(x)$ or otherwise, write the domain and range of $y=f(x)$ using interval notation.

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Question 13 (3 marks)

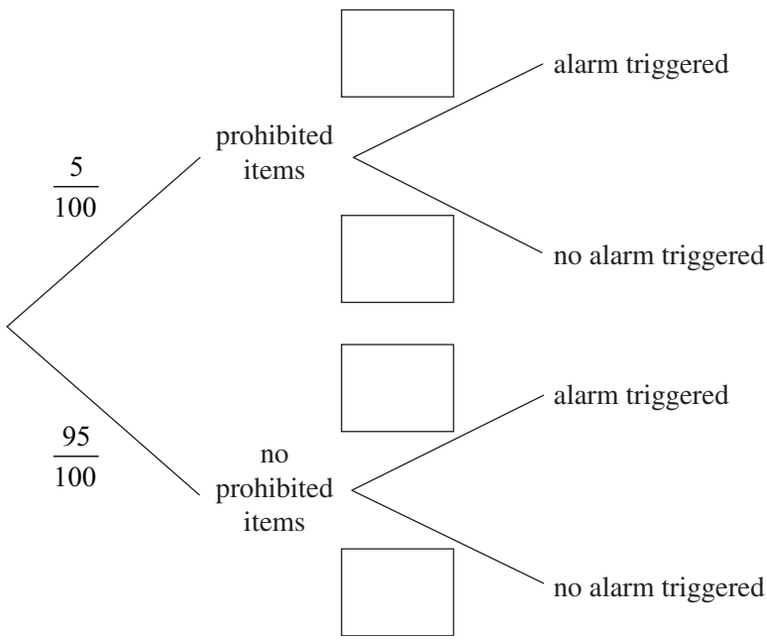
Prove that $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\text{cosec}A$. **3**

Question 14 (4 marks)

An airport scanner screens bags for prohibited items. An alarm is supposed to be triggered when a prohibited item is detected.

On a given day, 5% of bags contain prohibited items. If a bag contains prohibited items, there is a 95% chance that it triggers the alarm. If a bag does not contain prohibited items, there is a 2% chance that it triggers the alarm.

(a) Complete the probability tree using the information provided. **2**



(b) Given that a randomly selected bag triggers the alarm, what is the probability that it contains a prohibited item? **2**

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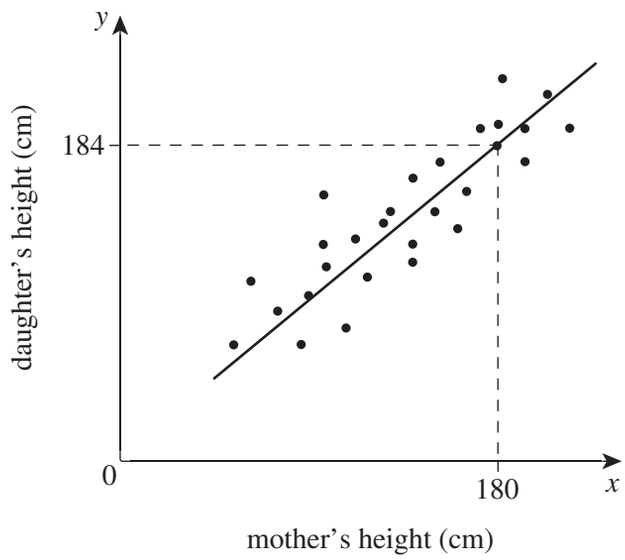
Question 15 (4 marks)

The heights of 30 mothers and daughters were recorded. The results are shown in the scatterplot. The least-squares regression line on the scatterplot passes through the point (180, 184).



(a) Katarina calculated that the correlation coefficient for this data set is $r = -2.5$. **2**

Without calculating the correlation coefficient, give TWO reasons why Katarina must be incorrect.

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(b) The equation of the least-squares regression line for this data set is in the form **2**

$$y = 0.8077x + A,$$

where A is a constant. Katarina is 160 cm tall.

Use the equation of the least-squares regression line to estimate the height of Katarina's mother. Give your answer correct to the nearest whole number.

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Question 16 (3 marks)

(a) Evaluate $f'(\pi)$ given that $f(x) = \dfrac{x^2}{\cos x}$. **3**

Question 17 (3 marks)

Let $y = x \ln x$.

(a) Show that $\dfrac{dy}{dx} = 1 + \ln x$. **1**

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(b) Hence, or otherwise, evaluate $\displaystyle\int_1^e \ln x \, dx$. **2**

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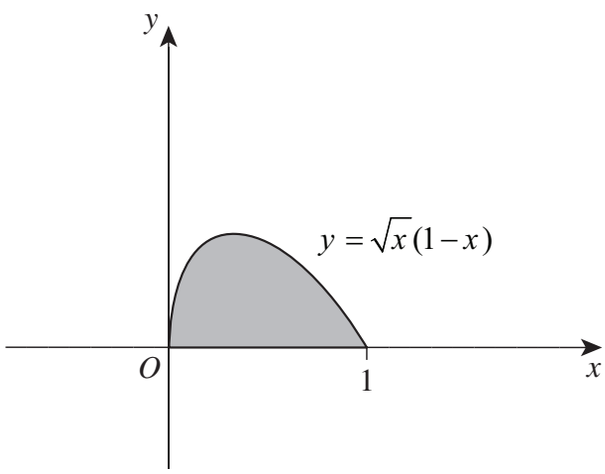
Question 18 (5 marks)

(a) Find the exact value of $\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2\left(\frac{\pi x}{2}\right) dx$. **3**

(b) Find the primitive function y given that $\frac{dy}{dx} = \frac{2x}{3x^2+1}$. **2**

Question 19 (3 marks)

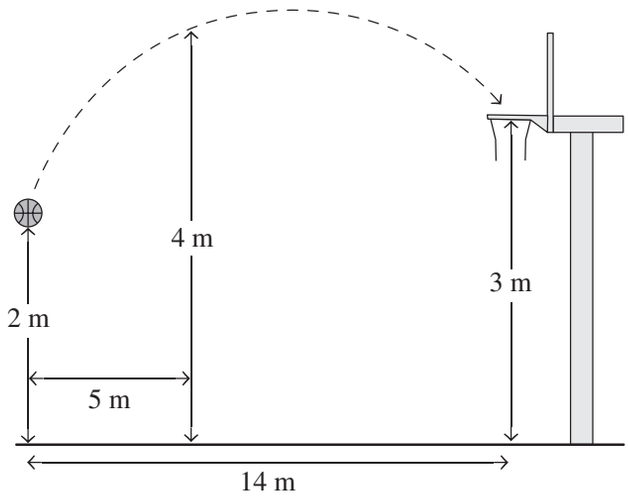
The graph of $y = \sqrt{x}\left(1-x\right)$ in the domain [0, 1] is shown. **3**



Find the area of the shaded region.

Question 20 (3 marks)

A basketball player shoots a ball from a height of 2 metres into a hoop 14 metres away. The hoop 3 metres from the ground. After moving 5 metres horizontally, the ball reaches a height of 4 metres from the ground. This information is shown in the diagram. **3**



NOT TO SCALE

The path of the ball can be modelled by the quadratic function

$$y = ax^2 + bx + c,$$

where y represents the height of the ball from the ground, x is the horizontal displacement from the basketball player, and a, b and c are constants.

Find the values of a, b and c that best model the path of the ball.

Question 21 (4 marks)

A company made a revenue of $200 000 in 2021. To maintain sustainable growth of the company, the executive team is considering two models to increase its revenue.

- Model A would increase the yearly revenue by 10% of the previous year's revenue.
- Model B would increase the yearly revenue by a fixed amount, $\$M$.

(a) If the company chooses model A, show that the company would make a revenue of $292 820 in 2025. **2**

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(b) If the company chooses model B, find the value of M such that the company's revenue is $292 820 in 2025. **2**

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

HSC Year 12 Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 22–31 (54 marks)

Instructions

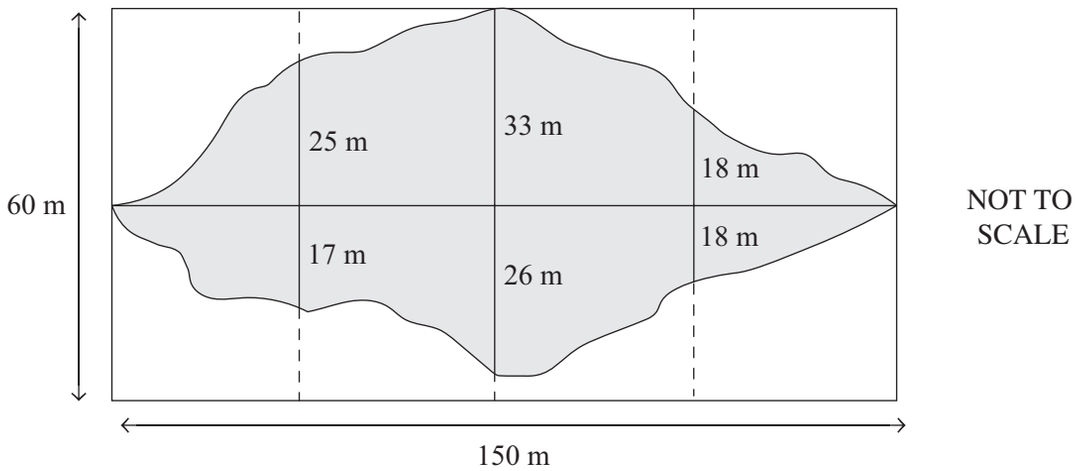
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 34–36 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 22 (2 marks)

Archaeologists are excavating a rectangular site with dimensions of 150 metres by 60 metres.

The shaded region indicates the portion of the site that has been excavated.



Use the trapezoidal rule to best approximate the total area that has been excavated. Give your answer correct to the nearest whole number. **2**

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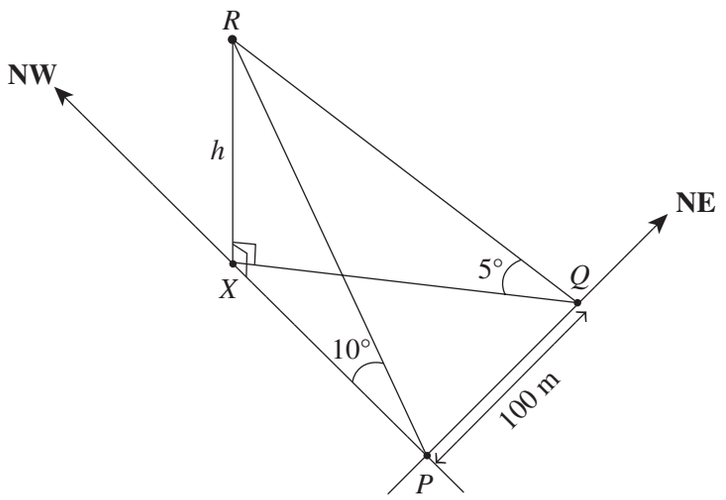
Question 23 (5 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q. A peak is located at point R, which is directly above point X. From point P, the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



NOT TO SCALE

(a) Show that $XP = h\cot 10°$ and find a similar expression for XQ. **2**

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(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

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Question 24 (7 marks)

On a given day, the height of the water in a river is modelled by the function

$$h(t) = 5 + 3\sin\left(\frac{\pi t}{4}\right),$$

where h is the height of the water, in metres, and t is the time, in hours, after 12 am.

(a) What is the height of the water at 12 am? **1**

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(b) Sketch the graph of $h(t) = 5 + 3\sin\left(\frac{\pi t}{4}\right)$ in the domain [0, 24]. **3**

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Question 24 continues on page 23

Question 24 (continued)

(c) A family decides to go on a picnic by the river from 12 pm to 2 pm. It is only safe to swim in the river if the height of the water is less than 4 metres. **3**

When is the earliest time the family can swim in the river after 12 pm? Give your answer correct to the nearest minute.

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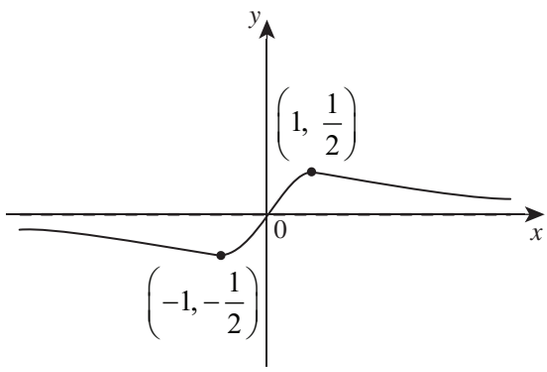
End of Question 24

Question 25 (4 marks)

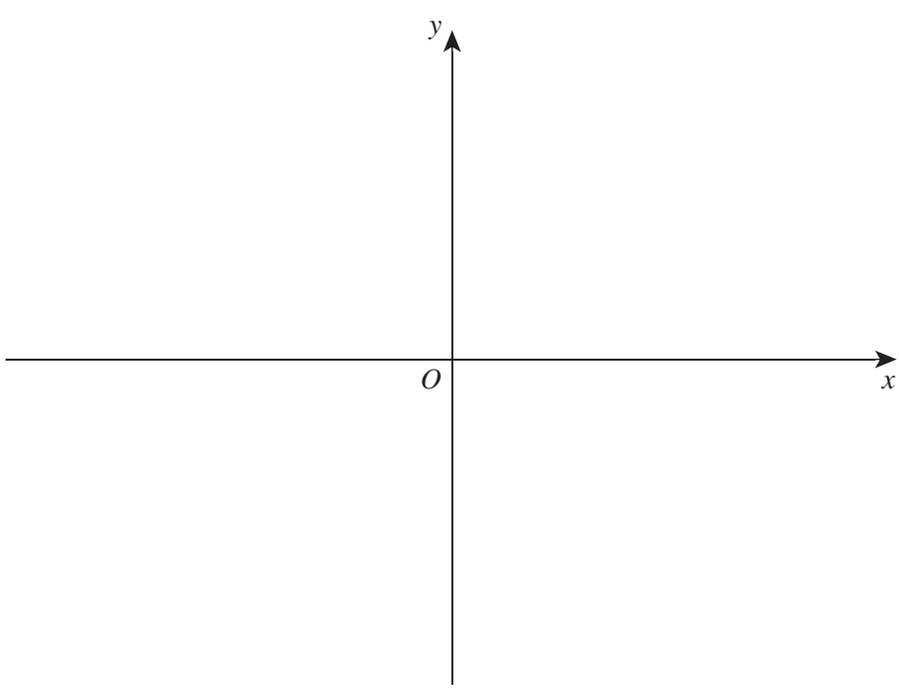
The graph of $y = f(x)$ in the domain $(-\infty, \infty)$ is shown. **4**



$y = f(x)$ has the following key features.

- The graph passes through the origin.
- There is a maximum turning point at $\left(1, \frac{1}{2}\right)$.
- There is a minimum turning point at $\left(-1, -\frac{1}{2}\right)$.
- The x-axis is a horizontal asymptote.

Sketch the graph of $y = -2f\left(\frac{x}{3}\right) + 1$, labelling any intercepts, asymptotes and turning points.



Question 26 (11 marks)

Consider the function $y = \dfrac{2x}{e^x}$.

It is known that as $x \to \infty$, $y \to 0$ and as $x \to -\infty = y \to \infty$

(a) Find the x- and y-intercepts. **1**

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(b) Find and classify any turning points on $y = \dfrac{2x}{e^x}$. **3**

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Question 26 continues on page 26

Question 26 (continued)

(c) Show that a point of inflection occurs at $\left(2,\ \frac{4}{e^2}\right)$. **3**

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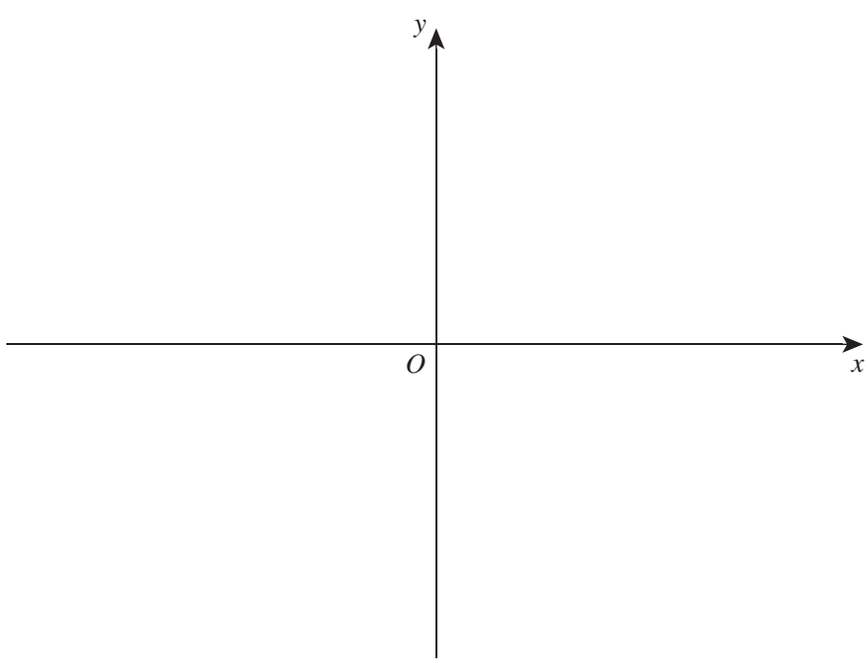
(d) Sketch the graph of $y = \frac{2x}{e^x}$, labelling all important features. **4**



End of Question 26

Question 27 (3 marks)

The time, in minutes, between the arrival of customers at an Automated Teller Machine (ATM) is represented by the variable, t, which is a continuous random variable modelled by the probability density function **3**

$$f(t) = \begin{cases} \frac{1}{3}e^{-\frac{t}{3}} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the median time between the arrival of customers at the ATM. Give your answer correct to the nearest minute.

Question 28 (5 marks)

A particle moves in a straight line such that its displacement is given by the function

$$x = 2 - \frac{3}{t+1},$$

where x is the particle's displacement from the origin, in metres, and t is the time, in seconds.

(a) What is the initial displacement of the particle? **1**

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(b) Find the expressions for the velocity and acceleration of the particle in terms of t. **2**

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(c) As t increases indefinitely, describe the motion of the particle with reference to its displacement and velocity. **2**

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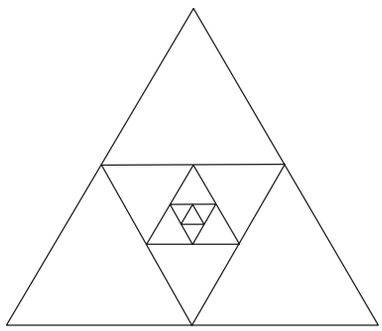
Question 29 (3 marks)

An equilateral triangle has a perimeter of p cm. The midpoints of each side are connected to form another triangle. This pattern can continue for an infinite number of times, as shown in the diagram.

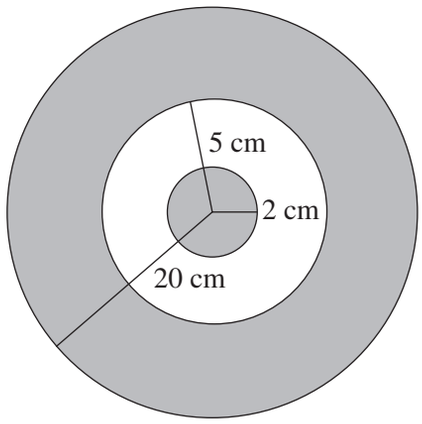


(a) Show that the perimeter of the nth triangle is $\frac{p}{2^{n-1}}$. **2**

(b) Find an expression in terms of p that represents the total perimeter of all the triangles. **1**

Question 30 (7 marks)

A simple dartboard consists of three sections: the centre circle, middle section and outer section. The sections have a radius of 2 cm, 5 cm and 20 cm respectively, as shown in the diagram.



NOT TO SCALE

After a player has thrown their dart, they are awarded a score according to the landing position of their dart on the dartboard. The table shows the scores awarded for each landing position.

Landing position	*Score*
centre circle	100
middle section	20
outer section	5

Assume that all darts thrown will hit the board and no darts will land on the lines.

(a) Paula, who is new to playing darts, throws a dart so that it lands somewhere on the board. Show that the probability that Paula's dart will land in the centre circle is $\frac{1}{100}$. **1**

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(b) Paula has two attempts at throwing a dart at the board. On one of her attempts, Paula's dart lands in the centre circle. On another attempt, her dart lands in the outer section. Show that the probability that this occurs, in any order, is $\frac{3}{160}$. **2**

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Question 30 continues on page 31

Question 30 (continued)

Let X be a discrete random variable that represents the number of points scored after two attempts at throwing a dart at the board. The probability distribution table is shown.

x	10	25	40	105	120	200
$P(X = x)$	$\frac{225}{256}$	$\frac{63}{640}$	$\frac{441}{160\ 000}$	$\frac{3}{160}$	$\frac{21}{20\ 000}$	$\frac{1}{10\ 000}$

An amateur dart player is throwing two darts to achieve a score. Assume the probability distribution table models the amateur player's likelihood of landing a dart in a specific section.

(c) Calculate the expected value of the probability distribution. **1**

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(d) Calculate the standard deviation of the probability distribution. Give your answer correct to two decimal places. **3**

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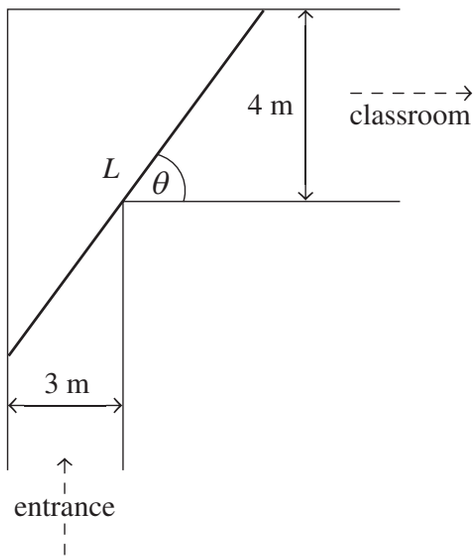
End of Question 30

Question 31 (7 marks)

A new whiteboard is being moved into a classroom. The whiteboard must be taken from the entrance, through the school's corridors and into the classroom. Two of the corridors are perpendicular to each other. The first corridor is 3 metres wide and the second corridor is 4 metres wide, as shown in the diagram. The length of the whiteboard is shown using L.



The whiteboard makes an angle θ to the horizontal on the corner of the corridors such that $0° < \theta < 90°$.

(a) Show that the length of the whiteboard, L, is $\dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$. **2**

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Question 31 continues on page 33

Question 31 (continued)

(b) What is the maximum possible length of the whiteboard that can be carried around the corner? **5**

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and exponential functions

$$\log_a a^x = x = a^{\log_a x}$$

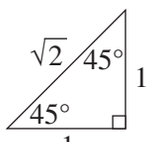
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}},\ \cos A = \frac{\text{adj}}{\text{hyp}},\ \tan A = \frac{\text{opp}}{\text{adj}}$$

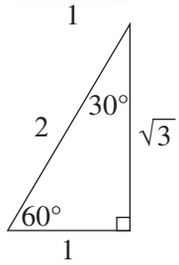
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\text{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

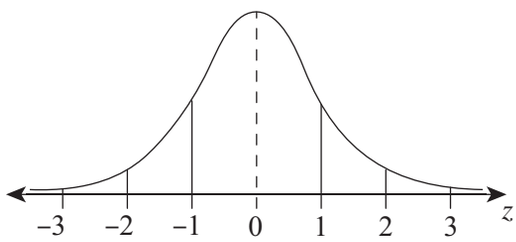
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 - 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)dx$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,\ 1,\ \ldots,\ n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f\left(x\right)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f\left(x_1\right)+\ldots+f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r\left(\cos\theta + i\sin\theta\right)$$
$$= re^{i\theta}$$

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Trial Examination 2022

HSC Year 12 Mathematics Advanced

DIRECTIONS:

Write your name in the space provided.
Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.
Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one** oval per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: _____

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I

MULTIPLE-CHOICE ANSWER SHEET

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II