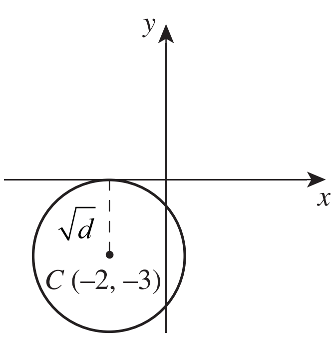
Neap

Trial Examination 2022

HSC Year 12 Mathematics Advanced

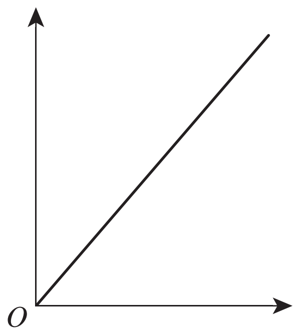
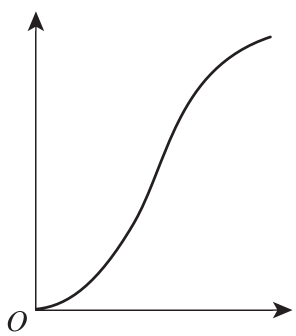
Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **D** **D** is correct. A many-to-many relationship is a relationship where one *x*-value maps to multiple *y*-values and one *y*-value is mapped from multiple *x*-values, as is the case with an ellipse. Hence, the diagram represents a many-to-many relationship. **A** and **C** are incorrect. These terms refer to functions. The ellipse clearly fails the vertical line test. Hence, the diagram does not represent a one-to-one or many-to-one relationship. **B** is incorrect. There are *y*-values that are mapped from multiple *x*-values. Therefore, the diagram does not represent a one-to-many relationship.	MA–F1 Working with Functions MA11–1 Band 3
Question 2 **D** $\mu = \frac{145+25}{2}$ $= 85$ Since it represents 99.7% of the area under the entire curve, the empirical rule states that the area bounded encompasses three standard deviations from the mean. $\sigma = \frac{85-25}{3}$ $= 20$ $\text{Var}(X) = \sigma^2$ $= 20^2$ $= 400$	MA–S3 Random Variables MA12–10 Band 3
Question 3 **D** The equation $(x+2)^2 + (y+3)^2 = d$ represents a circle with its centre at (–2, –3) and a radius of $\sqrt{d}$, as shown below.  For the *x*-axis to be a tangent to the circle, the diagram shows that the radius of the circle must be 3 units in length. $\sqrt{d} = 3$ $d = 9$	MA–F1 Working with Functions MA11–2 Bands 3–4

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 4 **B** From the table, the monthly repayment is \$836.44 for option 1 and \$807.14 for option 2. total amount of interest paid for option 1: $\text{interest} = \text{total amount} - \text{loan}$ $= (836.44 \times 12 \times 20) - 100\,000$ $= \$100\,745.60$ total amount of interest paid for option 2: $\text{interest} = \text{total amount} - \text{loan}$ $= (807.14 \times 12 \times 30) - 110\,000$ $= \$180\,570.40$ difference in interest: 180 570.40 – 100 745.60 = \$79 824.80	MA–M1 Modelling Financial Situations MA12–10 Bands 3–4
Question 5 **C** **Method 1:** $P(\text{sum} \geq 20) = P(9 \text{ and } 11) + P(9 \text{ and } 12)$ $= \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right)$ $= \frac{2}{9}$ **Method 2:** The total number of elements in the sample space is 3 × 3 = 9. There are only two cases where the sum will be greater than 20. They are obtaining either 9 and 11 or 9 and 12. $P(\text{sum} \geq 20) = \frac{2}{9}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 4
Question 6 **C** Given that $f(x) = \int_2^x \frac{1}{1+t^3}\,dt$, using the fundamental theorem of calculus gives: $f'(x) = \frac{d}{dx}\left[\int_2^x \frac{1}{1+t^3}\,dt\right]$ $= \frac{1}{1+x^3}$ Hence: $f'(2) = \frac{1}{1+2^3}$ $= \frac{1}{9}$	MA–C4 Integral Calculus MA12–3 Bands 5–6

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 **A** Recall that μ and $E(X)$ are equivalent. $E(X) = \Sigma xp(x)$ $= (1\times 0.05)+(2\times 0.15)+(5\times 0.4)+(6\times 0.2)+(8\times 0.2)$ $= 5.15$ $P(X > 5.15) = P(x=6)+P(x=8)$ $= 0.2+0.2$ $= 0.4$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 4
Question 8 **A** **Method 1:** Finding the points of intersection between the two curves gives: $px^2 = qx^2 + r$ $px^2 - qx^2 - r = 0$ $(p-q)x^2 - r = 0$ If there are no points of intersection, then $\Delta < 0$. $\Delta = b^2 - 4ac$ $= (0)^2 - 4(p-q)(-r)$ $= 4r(p-q)$ $4r(p-q) < 0$ $4r(p-q)$ is a negative value. If $r > 0$: $p - q < 0$ $\therefore p < q$ Hence, option **A** is the only option that represents these conditions. **Method 2:** **A** is correct. This can be found by checking each option by drawing two parabolas and assigning values for *p*, *r* and *q*. Substituting $p = 2$, $q = 3$ and $r = 1$ will describe a situation where both parabolas will not intersect. **B** is incorrect. The two parabolas will intersect if $p = 3$, $q = 2$ and $r = 1$. **C** is incorrect. The two parabolas will intersect if $p = -1$, $q = 1$ and $r = 0$. **D** is incorrect. The two parabolas will intersect if $p = 1$, $q = -1$ and $r = 0$.	MA–F1 Working with Functions MA11–9 Band 5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **C** **C** is correct. The box-plot indicates the data set has a symmetrical distribution. The curvature of the cumulative frequency diagram indicates that it is increasing at a decreasing rate up until the inflection point. After this, it increases at an increasing rate. The histogram in option **C** best represents this information as the frequencies are increasing at a decreasing rate as they move towards the centre, after which the frequencies increase at an increasing rate. **A** is incorrect. The cumulative frequency diagram for this histogram would be a diagonal line, as shown below. This is because the same frequency is added throughout the distribution.  **B** is incorrect. The cumulative frequency diagram for this histogram would increase at an increasing rate then increase at a decreasing rate, as shown below.  **D** is incorrect. This histogram does not represent a symmetrical distribution, as shown by the box-plot.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–10 Bands 5–6

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 10 **D** **Method 1:** **D** is correct. Examining the behaviour of $y=f\left(g\left(-x\right)\right)$ for both positive and negative values of x finds the following. • When $x=1$, $g(-x)=g(-1)>0$. Hence, $f\left(g\left(-1\right)\right)>0$, since $f(x)>0$ for $x>0$. • When $x=-1$, $g(-x)=g(1)>0$. Hence, $f\left(g(1)\right)>0$ for the same reason that $f(x)>0$ for $x>0$. • When $x=0$, $g(0)>0$. Hence, $f\left(g(0)\right)>0$. Therefore, the graph of $y=f\left(g\left(-x\right)\right)$ must be above the x-axis for all values of x. Examining the behaviour of $y=f\left(g\left(-x\right)\right)$ at the extremities finds that when $x\to-\infty$, $g\left(-x\right)=g(\infty)\to\infty$. Hence, $f\left(g\left(-x\right)\right)\to\infty$ since $f(x)\to\infty$ as $x\to\infty$. Therefore, the graph of $y=f\left(g\left(-x\right)\right)\to\infty$ as $x\to-\infty$. **A** and **C** are incorrect. The graphs exist below the x-axis. **B** is incorrect. When $x\to-\infty$, $f\left(g\left(-x\right)\right)\to 0$. **Method 2:** As $f(x)$ resembles $y=x^3$ and $g(x)$ resembles $y=e^x$: $f\left(g\left(-x\right)\right)=\left(e^{-x}\right)^3$ $=e^{-3x}$ Therefore, This graph should behave like an exponential function in the form $y=e^{-3x}$. Option **D** best represents this information.	MA–F1 Working with Functions MA11–9, 12–10 Bands 5–6

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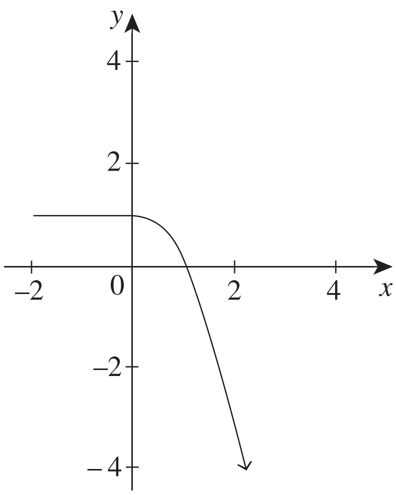
SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
$\lvert 2x-3 \rvert = 4$ $2x-3=4$ $2x=7$ $x=\frac{7}{2}$ $-(2x-3)=4$ $2x-3=-4$ $2x=-1$ $x=-\frac{1}{2}$ $\therefore x=\frac{7}{2}, \frac{1}{2}$	MA–F1 Working with Functions MA11–2 Band 3 • Provides the correct solutions . . . . 2 • Develops a linear equation AND provides its correct solution. 1
Question 12	
A sketch of $y=f(x)$ is shown.  From the graph, the domain is [–2, ∞) and the range is (–∞, 1].	MA–F1 Working with Functions MA11–2, 11–9 Bands 3–4 • Provides the correct domain AND range using interval notation . . . . 2 • Provides the correct domain OR range using interval notation 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	

$$\text{LHS} = \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$$
$$= \frac{\sin^2 A}{\sin A\left(1+\cos A\right)} + \frac{\left(1+\cos A\right)^2}{\sin A\left(1+\cos A\right)}$$
$$= \frac{\sin^2 A + \left(1+\cos A\right)^2}{\sin A\left(1+\cos A\right)}$$
$$= \frac{\sin^2 A + \left(1+2\cos A + \cos^2 A\right)}{\sin A\left(1+\cos A\right)}$$
$$= \frac{2+2\cos A}{\sin A\left(1+\cos A\right)} \qquad \left(\text{since } \sin^2 A + \cos^2 A = 1\right)$$
$$= \frac{2\left(1+\cos A\right)}{\sin A\left(1+\cos A\right)}$$
$$= \frac{2}{\sin A}$$
$$= 2\operatorname{cosec}A$$
$$= \text{RHS}$$

MA–T2 Trigonometric Functions and Identities
MA11–4 Bands 3–4

- Provides the correct solution 3

- Combines the fractions together AND applies trigonometric identities to assist with the proof of the expression.

OR

- Equivalent merit 2

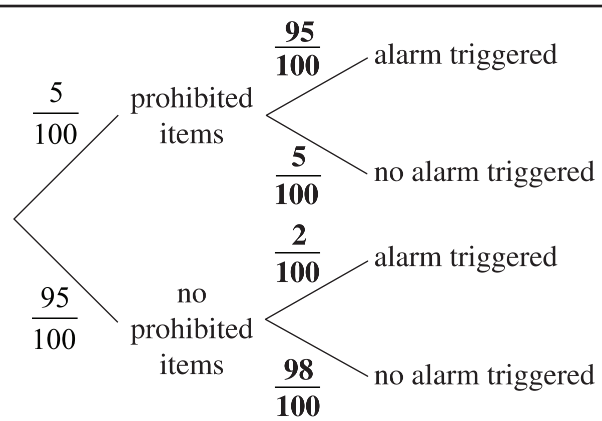
- Combines the fractions together.

OR

- Equivalent merit 1

Question 14

(a)



MA–S1 Probability and Discrete Probability Distributions
MA11–7 Bands 3–4

- Provides FOUR correct values . . . 2

- Provides at least TWO correct values. 1

(b)
$$P\left(\text{prohibited} \mid \text{alarm}\right) = \frac{P\left(\text{prohibited} \cap \text{alarm}\right)}{P\left(\text{alarm}\right)}$$
$$= \frac{\left(\frac{5}{100}\times\frac{95}{100}\right)}{\left(\frac{5}{100}\times\frac{95}{100}\right)+\left(\frac{95}{100}\times\frac{2}{100}\right)}$$
$$= \frac{5}{7}$$

MA–S1 Probability and Discrete Probability Distributions
MA11–8 Band 4

- Provides the correct solution 2

- Indicates the use of the conditional probability formula 1

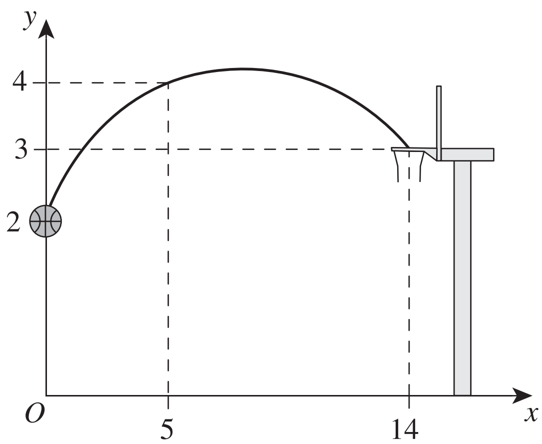
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 15	
(a) Katarina is incorrect because a correlation coefficient is a value between $-ü \le r \le$ A value of $r = -2.5$ exists outside this restriction. She is also incorrect because the scatter plot indicates that there is a positive correlation. Hence, the correlation coefficient should have a positive value between $0 < r < 1$.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–10 Bands 3–4 • Provides TWO valid reasons . . . . . 2 • Provides ONE valid reason 1
(b) $y = 0.8077x + A$ Using the point (180, 184): $184 = 0.8077(180) + A$ $184 = 145.386 + A$ $A = 38.614$ Hence, the least-squares regression line is $y = 0.8077x + 38.614$. When $y = 160$: $160 = 0.8077x + 38.614$ $x = 150.285...$ Therefore, Katarina's mother is approximately 150 cm tall.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–9 Bands 3–4 • Provides the correct solution . . . . . 2 • Finds the value of A 1
Question 16	
(a) $f(x) = \dfrac{x^2}{\cos x}$ $f'(x) = \dfrac{(\cos x)(2x) - (x^2)(-\sin x)}{\cos^2 x}$ $= \dfrac{x(2\cos x + x\sin x)}{\cos^2 x}$ $f'(\pi) = \dfrac{\pi(2\cos\pi + \pi\sin\pi)}{\cos^2\pi}$ $= \dfrac{\pi(-2+0)}{(-1)^2}$ $= -2\pi$	MA–C2 Differential Calculus MA12–6 Bands 3–4 • Provides the correct solution . . . . . 3 • Substitutes $x = \pi$ into $f'(x)$ . . . . . 2 • Uses the quotient rule to differentiate $f(x)$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 17	
(a) $y = x\ln x$ $\frac{dy}{dx} = (x)\left(\frac{1}{x}\right) + (\ln x)(1)$ $= 1 + \ln x$	MA–C3 Applications of Differentiation MA12–3 Bands 3–4 • Provides the correct solution 1
(b) If ü = ü $\frac{dy}{dx} = 1 + \ln x$ (from part (a)). Using the fundamental theorem of calculus: $\int_1^e (1+\ln x)\,dx = [x\ln x]_1^e$ $\int_1^e 1\,dx + \int_1^e \ln x\,dx = [x\ln x]_1^e$ $\int_1^e \ln x\,dx = [x\ln x]_1^e - \int_1^e 1\,dx$ $= [x\ln x]_1^e - [x]_1^e$ $= (e\ln e - 1\ln 1) - (e-1)$ $= e - (e-1)$ $= 1$ *Note: Consequential on answer to **Question 17(a)**.*	MA–C4 Integral Calculus MA12–10 Band 4 • Provides the correct solution . . . . . 2 • Applies the fundamental theorem of calculus to the solution found in part (a) 1
Question 18	
(a) $I = \int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2\left(\frac{\pi x}{2}\right)$ $= \left[\frac{1}{\left(\frac{\pi}{2}\right)}\tan\left(\frac{\pi x}{2}\right)\right]_{\frac{1}{3}}^{\frac{1}{2}}$ $= \frac{2}{\pi}\left[\tan\left(\frac{\pi x}{2}\right)\right]_{\frac{1}{3}}^{\frac{1}{2}}$ $= \frac{2}{\pi}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right)$ $= \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{3}}\right)$	MA–C4 Integral Calculus MA12–7 Bands 3–4 • Provides the correct solution . . . . . 3 • Substitutes the boundaries AND simplifies the anti-derivative . . . . . 2 • Finds the anti-derivative 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $\dfrac{dy}{dx}=\dfrac{2x}{3x^2+1}$ $y=\displaystyle\int\frac{2x}{3x^2+1}dx$ $=\dfrac{1}{3}\displaystyle\int\frac{6x}{3x^2+1}dx$ $=\dfrac{1}{3}\ln\lvert 3x^2+1\rvert+C$	MA–C4 Integral Calculus MA12–3 Bands 3–4 • Provides the correct solution . . . . . 2 • Manipulates the integrand into the form $\dfrac{f'(x)}{f(x)}$ 1
Question 19	
$A=\displaystyle\int_0^1\sqrt{x}\left(1-x\right)dx$ $=\displaystyle\int_0^1\sqrt{x}-x\sqrt{x}\,dx$ $=\displaystyle\int_0^1 x^{\frac{1}{2}}-x^{\frac{3}{2}}\,dx$ $=\left[\dfrac{2}{3}x^{\frac{3}{2}}-\dfrac{2}{5}x^{\frac{5}{2}}\right]_0^1$ $=\left(\dfrac{2}{3}-\dfrac{2}{5}\right)-\left(0-0\right)$ $=\dfrac{4}{15}$ units^2	MA–C4 Integral Calculus MA12–10 Band 4 • Provides the correct solution . . . . . 3 • Finds the anti-derivative . . . . 2 • Expresses the integrand in the form x^n 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 20	

The parabola has the equation $y = ax^2 + bx + c$. The following diagram shows a cartesian plane assigned to the diagram.



The ball passes through the points (0, 2), (5, 4) and (14, 3).

Substituting (0, 2) gives:

$$2 = a(0)^2 + b(0) + c$$
$$c = 2$$

Hence, the equation is now represented as $y = ax^2 + bx + 2$.

Substituting (5, 4) gives:

$$4 = a(5)^2 + b(5) + 2$$
$$2 = 25a + 5b$$
$$a = \frac{2-5b}{25} \quad \text{(equation 1)}$$

Substitution (14, 3) gives:

$$3 = a(14)^2 + b(14) + 2$$
$$1 = 196a + 14b \quad \text{(equation 2)}$$

Substituting equation 1 into equation 2 gives:

$$1 = 196\left(\frac{2-5b}{25}\right) + 14b$$
$$25 = 196(2-5b) + 350b$$
$$25 = 392 - 980b + 350b$$
$$-367 = -630b$$
$$b = \frac{367}{630}$$

(continues on next page)

MA–F1 Working with Functions
MA11–8, 11–9 Band 4

- Provides the correct solution 3
- Finds the value of c and develops simultaneous equations to solve for a and b 2
- Finds the value of c 1

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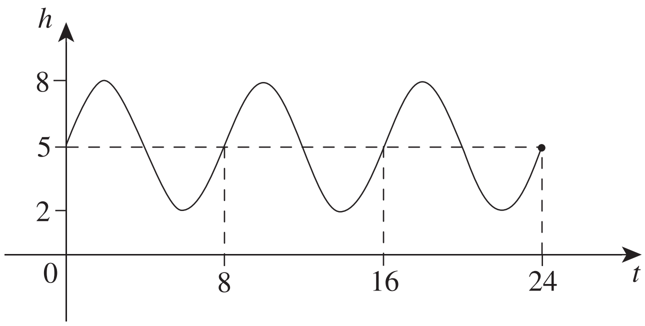
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(continued) Substituting the value of b into equation 1 gives: $a = \dfrac{2-5\left(\dfrac{367}{630}\right)}{25}$ $= -\dfrac{23}{630}$ Therefore, $y = -\dfrac{23}{630}x^2 + \dfrac{367}{630}x + 2$.	
Question 21	
(a) **Method 1:** Model A represents a geometric sequence where $a = 200\,000$ and $r = \dfrac{110}{100} = 1.1$. Hence, the general term is $T_n = 200\,000(1.1)^{n-1}$. For the year 2025, let $n = 5$. $T_5 = 200\,000(1.1)^{5-1}$ $= 292\,820$ **Method 2:** $A = P(1+r)^n$ $= 200\,000(1+0.1)^4$ $= 292\,820$	MA–M1 Modelling Financial Situations MA12–4 Bands 3–4 • Provides the correct solution . . . . . 2 • Finds the general term T_n. OR • Equivalent merit 1
(b) Model B represents an arithmetic sequence where $a = 200\,000$ and $d = M$. Hence, the general term is $T_n = 200\,000 + (n-1)M$. Since $T_5 = 292\,820$: $292\,820 = 200\,000 + (5-1)M$ $92\,820 = 4M$ $M = \$23\,205$	MA–M1 Modelling Financial Situations MA12–2 Band 4 • Provides the correct solution . . . . . 2 • Finds the general term T_n. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 22	
The shaded region is described by the table of values below. x: 0, 37.5, 75, 112.5, 150 y: 0, 42, 59, 36, 0 Applying the formula for the trapezoidal rule using $n = 4$ to find the approximate area: $A \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+...+f(x_{n-1})]\}$ $\approx \frac{150-0}{2(4)}\{0+0+2(42+59+36)\}$ $\approx 5137.5 \text{ m}^2$	MA–C4 Integral Calculus MA12–3 Bands 3–4 • Provides the correct solution . . . . . 2 • Provides a correct table of values with function values. OR • Equivalent merit 1
Question 23	
(a) As ΔRPX is a right-angled triangle: $\tan 10° = \frac{RX}{XP}$ $\tan 10° = \frac{h}{XP}$ $XP = \frac{h}{\tan 10°}$ $\therefore XP = h\cot 10°$ As ΔRQX is a right-angled triangle: $\tan 5° = \frac{h}{XQ}$ $XQ = h\cot 25°$	MA–T1 Trigonometry and Measure of Angles MA11–3, 12–1 Bands 3–4 • Shows that $XP = h\cot 10°$. AND • Finds the expression for XQ . . . . . 2 • Shows that $XP = h\cot 10°$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) Using Pythagoras' theorem in ΔXPQ: $XP^2 + PQ^2 = XQ^2$ $\left(h\cot 10°\right) + \left(100\right)^2 = \left(h\cot 5°\right)^2$ $h^2\cot^2 10° + 10\,000 = h^2\cot^2 5°$ $10\,000 = h^2\cot^2 5° - h^2\cot 10°$ $10\,000 = h^2\left(\cot^2 5° - \cot^2 10°\right)$ $h^2 = \dfrac{10\,000}{\cot^2 5° - \cot^2 10°}$ $h = \sqrt{\dfrac{10\,000}{\cot^2 5° - \cot^2 10°}}$ $= 10$ m	MA–T1 Trigonometry and Measure of Angles MA12–9 Bands 3–4 • Provides the correct solution . . . . . 3 • Makes h^2 the subject . . . . 2 • Applies Pythagoras' theorem to ΔXPQ 1
Question 24	
(a) When $t = 0$: $h = 5 + 3\sin\left(\dfrac{\pi}{4} \times 0\right)$ $= 5 + 3\sin 0$ $= 5$ m	MA–T3 Trigonometric Functions and Graphs MA12–5 Bands 2–3 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) Finding the period: $T = \frac{2\pi}{n}$ $= \frac{2\pi}{\left(\frac{\pi}{4}\right)}$ $= 8$ The amplitude is 3. Hence, the maximum value is $5 + 3 = 8$ and the minimum value is $5 - 3 = 2$.  	MA–T3 Trigonometric Functions and Graphs MA12–5 Band 4 • Sketches a graph that shows all THREE of: – the period – the maximum and minimum values – the correct graph shape. . . . . . . 3 • Any TWO of the above points. . . . 2 • Any ONE of the above points 1
(c) When $h = 4$: $4 = 5 + 3\sin\left(\frac{\pi}{4}t\right)$ $-\frac{1}{3} = \sin\left(\frac{\pi}{4}t\right)$ $\frac{\pi}{4}t = \sin^{-1}\left(-\frac{1}{3}\right)$ $\frac{\pi}{4}t = (\pi + 0.3398),\ (2\pi - 0.3398),\ (3\pi + 0.3398),\ (4\pi - 0.3398),\ \ldots$ $t = \frac{\pi}{4}(\pi + 0.3398),\ \frac{\pi}{4}(2\pi - 0.3398),\ \frac{\pi}{4}(3\pi + 0.3398),\ \frac{\pi}{4}(4\pi - 0.3398),\ \ldots$ $= 4.433$ hours, 5.851 hours, 12.43 hours, 15.57 hours, … Since the family will be by the river between 12 pm to 2 pm, the solution $t = 12.43$ indicates the first time the family is safe to swim in the river. As 12.43 hours = 12 hours and 26 minutes, the earliest time the family can swim in the river is 12:26 pm.	MA–T3 Trigonometric Functions and Graphs MA12–1, 12–5, 12–10 Bands 4–5 • Provides the correct solution . . . . . 3 • Solves the trigonometric equation for possible values of t. . . . . 2 • Substitutes $h = 4$ to develop a trigonometric equation 1

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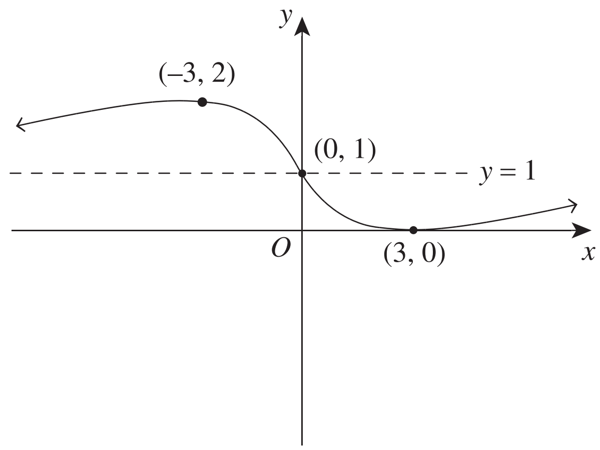
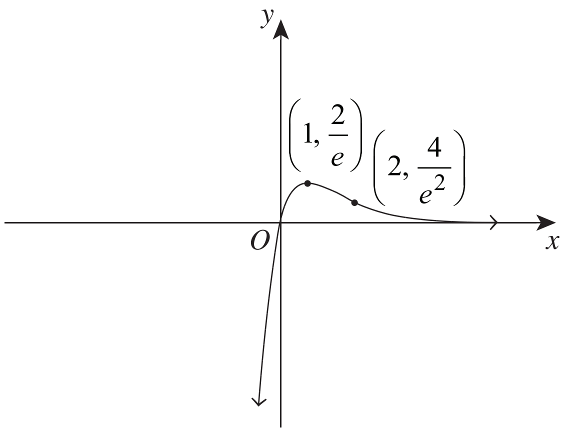
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 25	
(a) 	MA–F2 Graphing Techniques MA12–1 Bands 4–5 • Sketches a graph that shows all FOUR of: – a reflection about the x-axis – a vertical dilation with scale factor of 2 – a horizontal dilation with scale factor of 3 – a vertical translation of one unit upwards . . . . 4 • Any THREE of the above points . . 3 • Any TWO of the above points. . . . 2 • Any ONE of the above points 1
Question 26	
(a) To find the x-intercepts, let $y = 0$. $0 = \frac{2x}{e^x}$ $0 = 2x$ $x = 0$ Therefore, the x-intercept is (0, 0). To find the y-intercepts, let $x = 0$. $y = \frac{2(0)}{e^0}$ $= 0$ Therefore, the y-intercept is (0, 0). Hence, $y = \frac{2x}{e^x}$ only has one intercept at the origin.	MA–C3 Applications of Differentiation MA12–3 Bands 2–3 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $y=\frac{2x}{e^x}$ $\frac{dy}{dx}=\frac{(e^x)(2)-(2x)(e^x)}{(e^x)^2}$ $=\frac{2e^x(1-x)}{e^{2x}}$ $=\frac{2(1-x)}{e^x}$ For stationary points, $\frac{dy}{dx}=0$. $0=\frac{2(1-x)}{e^x}$ $1-x=0$ $x=1$ When $x = 1$: $y=\frac{2(1)}{e^1}$ $=\frac{2}{e}$ Determining the nature of the stationary point using the first derivative table gives: (see table below) Hence, the turning point $\left(1, \frac{2}{e}\right)$ is a maximum turning point.	MA–C3 Applications of Differentiation MA12–6 Bands 4–5 • Tests and determines the nature of the stationary point $\left(1, \frac{2}{e}\right)$ . . . . . . 3 • Finds the stationary point at $\left(1, \frac{2}{e}\right)$ . . . . 2 • Finds the derivative 1

x	0	1	2
$\frac{dy}{dx}$	2 (positive)	0	$-\frac{2}{e^2}$ (negative)

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $\frac{dy}{dx}=\frac{2(1-x)}{e^x}$ $\frac{d^2y}{dx^2}=2\left[\frac{(e^x)(-1)-(1-x)(e^x)}{e^{2x}}\right]$ $=2\left(\frac{xe^x-2e^x}{e^{2x}}\right)$ $=2\left(\frac{x-2}{e^x}\right)$ For possible points of inflection, $\frac{d^2y}{dx^2}=0$. $0=2\left(\frac{x-2}{e^x}\right)$ $x-2=0$ $x=2$ When $x=2$: $y=\frac{2(2)}{e^2}$ $=\frac{4}{e^2}$ Testing the point of inflection using the second derivative table gives: (see table below) As there is a change in concavity, $\left(2,\frac{4}{e^2}\right)$ is a point of inflection.	MA–C3 Applications of Differentiation MA12–10 Bands 4–5 • Provides the correct solution . . . . . 3 • Finds a possible point of inflection occurs at $\left(2,\frac{4}{e^2}\right)$ . . . . 2 • Finds the second derivative 1

x	1	2	3
$\frac{d^2y}{dx^2}$	$-\frac{2}{e}$ (negative)	0	$\frac{2}{e^3}$ (positive)

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) 	MA–C3 Applications of Differentiation MA12–10 Band 4 • Sketches a graph that shows all FOUR of: – the intercepts – the maximum turning point – the point of inflection – the correct behaviour as $x \to \pm\infty$ . . . . 4 • Any THREE of the above points . . 3 • Any TWO of the above points. . . . 2 • Any ONE of the above points 1
Question 27	
Let $F(x)$ be the cumulative distribution function. $F(x) = \int_0^x \frac{1}{3} e^{-\frac{t}{3}}\, dt$ $= \frac{1}{\not{3}}\left[-\not{3} e^{-\frac{t}{3}}\right]_0^x$ $= -\left[e^{-\frac{x}{3}} - e^0\right]$ $= -\left(e^{-\frac{x}{3}} - 1\right)$ $= 1 - e^{-\frac{x}{3}}$ For the median time taken, let $F(x) = 0.5$. $0.5 = 1 - e^{-\frac{x}{3}}$ $-0.5 = -e^{-\frac{x}{3}}$ $0.5 = e^{-\frac{x}{3}}$ $\ln 0.5 = \ln\left(e^{-\frac{x}{3}}\right)$ $\ln 0.5 = -\frac{x}{3}$ $x = -3\ln 0.5$ $= 2.079...$ Therefore, the median time taken is 2 minutes.	MA–S3 Random Variables MA12–8 Band 4 • Provides the correct solution . . . . . 3 • Makes valid progress in solving $F(x) = 0.5$ for the median time. . . . 2 • Finds the cumulative distribution function 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
(a) When $t = 0$: $x = \frac{2(0)-1}{0+1} = -1$ Hence, the particle is initially one metre to the left of the origin.	MA–C1 Introduction to Differentiation MA11–5 Band 3 • Provides the correct solution 1
(b) $x = 2 - 3(t+1)^{-1}$ $v = 3(t+1)^{-2} = \frac{3}{(t+1)^2}$ $a = -6(t+1)^{-3} = \frac{-6}{(t+1)^3}$	MA–C1 Introduction to Differentiation MA11–5 Bands 3–4 • Finds the expression for v AND a . . . . 2 • Finds the expression for v OR a 1
(c) As $t \to \infty,\ x \to 2 - 0 = 2.$ Hence, the particle approaches $x = 2$ m. As $t \to \infty, v \to 0.$ Hence, the particle's velocity is slowing down and approaching 0 m s^{-1}. As t increases indefinitely, the particle is approaching $x = 2$ m with decreasing speed.	MA–C1 Introduction to Differentiation MA11–9 Bands 4–5 • Describes the particle's displacement AND velocity as $t \to \infty$ . . . . 2 • Describes the particle's displacement OR velocity as $t \to \infty$ 1
Question 29	
(a) The perimeters of the triangles form a geometric sequence where $a = p$ and $r = \frac{1}{2}$. Let p_n represent the perimeter of the nth triangle. $p_n = ar^{n-1} = p\left(\frac{1}{2}\right)^{n-1} = p\left(\frac{1^{n-1}}{2^{n-1}}\right) = \frac{p}{2^{n-1}}$ $\left(\text{since } 1^{n-1} = 1\right)$	MA–M1 Modelling Financial Situations MA12–4 Bands 4–5 • Provides the correct solution . . . . . 2 • Finds an expression for p_n. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $S_\infty = \frac{a}{1-r}$ $= \frac{p}{1-\frac{1}{2}}$ $= \frac{p}{\frac{1}{2}}$ $= 2p$	MA–M1 Modelling Financial Situations MA12–4 Bands 3–4 • Provides the correct solution 1
Question 30	
(a) $P(\text{centre}) = \frac{\pi(2)^2}{\pi(20)^2}$ $= \frac{4}{400}$ $= \frac{1}{100}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 4 • Provides the correct solution 1
(b) P(centre circle and outer section in any order) $= 2\times\left(\frac{1}{100}\times\frac{\pi(20)^2-\pi(5)^2}{\pi(20)^2}\right)$ $= 2\times\left(\frac{1}{100}\times\frac{15}{16}\right)$ $= \frac{3}{160}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 4 • Provides the correct solution . . . . . 2 ——— • Finds the probability of a dart landing in the outer section 1
(c) $E(X) = \Sigma xp(x)$ $= \left(10\times\frac{225}{256}\right)+\left(25\times\frac{63}{640}\right)$ $+\left(40\times\frac{441}{160\,000}\right)+\left(105\times\frac{3}{160}\right)$ $+\left(120\times\frac{21}{20\,000}\right)+\left(200\times\frac{1}{10\,000}\right)$ $= 13.475$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 3 • Provides the correct solution 1

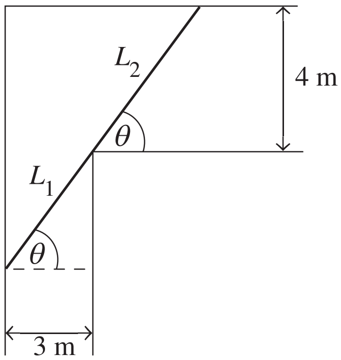
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d)	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 4–5 • Provides the correct solution . . . . . 3 • Provides the correct value for Var(X). . . . . 2 • Finds the values for the $x^2p(x)$ column 1

x	$P(X=x)$	$x^2p(x)$
10	$\frac{225}{256}$	$\frac{5625}{64}$
25	$\frac{63}{640}$	$\frac{7875}{128}$
40	$\frac{441}{160\,000}$	$\frac{441}{100}$
105	$\frac{3}{160}$	$\frac{6615}{32}$
120	$\frac{21}{20\,000}$	$\frac{378}{25}$
200	$\frac{1}{10\,000}$	4

$$\begin{aligned}\text{Var}(X) &= \Sigma x^2p(x) - \mu^2\\ &= \left(\frac{5625}{64}\right)+\left(\frac{7875}{128}\right)+\left(\frac{441}{100}\right)+\left(\frac{6615}{32}\right)\\ &\quad+\left(\frac{378}{25}\right)+(4)-13.475^2\\ &= 198.087\ldots\end{aligned}$$

For standard deviation:

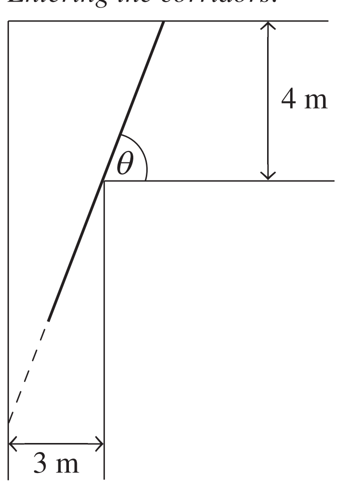
$$\begin{aligned}\sigma &= \sqrt{\text{Var}(X)}\\ &= \sqrt{198.087\ldots}\\ &= 14.07 \text{ (to two decimal places)}\end{aligned}$$

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 31	
(a) The length of the whiteboard, L, can be found by dividing it into two separate parts, L_1 and L_2.  As $\cos\theta = \dfrac{3}{L_1}$, $L_1 = \dfrac{3}{\cos\theta}$. As $\sin\theta = \dfrac{4}{L_2}$, $L_2 = \dfrac{4}{\sin\theta}$. Therefore, the length of the whiteboard is: $L = L_1 + L_2$ $= \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$	MA–C3 Applications of Differentiation MA12–3 Bands 4–5 • Provides the correct solution . . . . . 2 • Makes valid progress to show the expression for L 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) To find the angle that minimises the function $L = \frac{3}{\cos\theta} + \frac{4}{\sin\theta}$: $L = \frac{3}{\cos\theta} + \frac{4}{\sin\theta}$ $= 3(\cos\theta)^{-1} + 4(\sin\theta)^{-1}$ $\frac{dL}{d\theta} = -3(\cos\theta)^{-2}(-\sin\theta) - 4(\sin\theta)^{-2}(\cos\theta)$ $= \frac{3\sin\theta}{\cos^2\theta} - \frac{4\cos\theta}{\sin^2\theta}$ $= \frac{3\sin^3\theta - 4\cos^3\theta}{\cos^2\theta\sin^2\theta}$ For stationary points, $\frac{dL}{d\theta} = 0$. $0 = \frac{3\sin^3\theta - 4\cos^3\theta}{\cos^2\theta\sin^2\theta}$ $0 = 3\sin^3\theta - 4\cos^3\theta$ $4\cos^3\theta = 3\sin^3\theta$ $\frac{4}{3} = \tan^3\theta$ $\tan\theta = \left(\frac{4}{3}\right)^{\frac{1}{3}}$ $\theta = \tan^{-1}\left[\left(\frac{4}{3}\right)^{\frac{1}{3}}\right]$ $= 0.833\ldots$ Converting into degrees gives: $0.833 \times \frac{180}{\pi} = 47.74°$ Determining the nature of the stationary point using the first derivative test gives:	MA–C3 Applications of Differentiation MA12–9, 12–10 Band 6 • Provides the correct solution . . . . . 5 • Tests AND shows that $\theta = 47.47°$ is a minimum turning point . . . . 4 • Shows that $\theta = 47.47°$ is a stationary point . . . . 3 • Solves $\frac{dL}{d\theta} = 0$ . . . . 2 • Finds an expression for $\frac{dL}{d\theta}$ 1

θ	47°	47. 74°	48°
$\frac{dL}{d\theta}$	–0.383 (negative)	0	0.133 (positive)

(continues on next page)

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(continued) Hence, $\theta = 47.74°$ is a minimum turning point. For this reason, the maximum possible length of L occurs when $\theta = 47.74°$. When $\theta = 47.74°$: $L = \dfrac{3}{\cos 47.74°} + \dfrac{4}{\sin 47.74°}$ ≈ 9.86 Therefore, the maximum possible length of the whiteboard is 9.86 metres. *Note: Accept the final answer rounded up or down. No marks are awarded for correct rounding.* *To find the maximum length that the whiteboard can be carried around the corner, θ must be found that minimises $L = \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$.* *Notice that the whiteboard will form an angle θ such that $0° < \theta < 90°$.* *As the whiteboard enters from the bottom corridor, the angle θ will start towards its upper bound, 90°. As the whiteboard moves around the corner, θ will decrease towards its lower bound, 0°.* *Entering the corridors:*  (continues on next page)	

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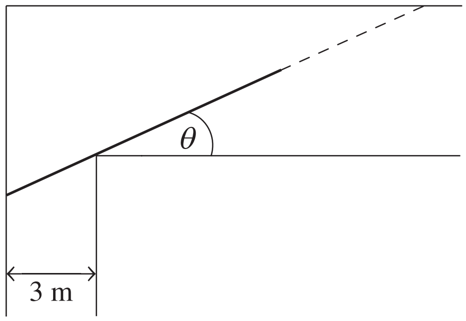
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide

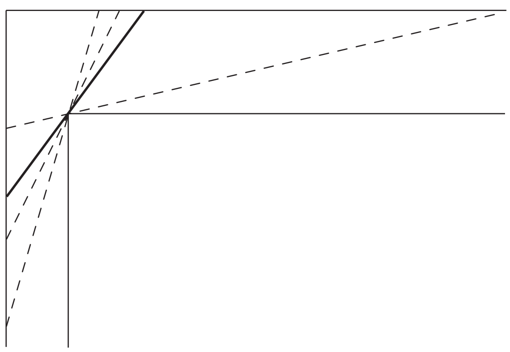
(continued)

Moving around the corner:



When θ is close to either the lower or upper bound, the length of the whiteboard is infinitely long. This is also represented when θ is considered as 90° or 0° in the expression $L = \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$. *That is, when* $\theta \to 90°$ *or* $0°$, $L \to \infty$. *However, in these situations, the whiteboard will be too long to make it around the corner.*

Hence, the response should find the value of θ such that L is optimally long enough to still be able to turn around the corner.



Notice in the diagram that the value of θ for the whiteboard turning the corner will give the shortest possible length for L.

Hence, to find the maximum possible length of the whiteboard such that it can turn around the corner, the angle that minimises the function $L = \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$ *must be found.*