Neap

Final Examination 2022

NSW Year 11 Mathematics Extension 1

General Instructions	
	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 70	SECTION I – 10 marks (pages 2–4) • Attempt Questions 1–10 • Allow about 15 minutes for this section SECTION II – 60 marks (pages 5–9) • Attempt Questions 11–14 • Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is the solution to $|2x-1|>5$?

A. $-2<x<3$

B. $x<-2,\ x>3$

C. $-3<x<2$

D. $x<-3,\ x>2$

2 The polynomial $4x^3-2x^2+8x-1=0$ has zeros α, β and γ.
What is the value of $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$?

A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

3 A curve is defined by the equations $x=2\cos\theta-1$ and $y=4\sin\theta+1$.
What is the Cartesian equation of the curve?

A. $2(x+1)^2+(y-1)^2=4$

B. $(x+1)^2+2(y-1)^2=4$

C. $4(x+1)^2+(y-1)^2=16$

D. $(x+1)^2+4(y-1)^2=16$

4 The soccer coach at a school has decided that the senior soccer team will consist of eight Year 12 students and six Year 11 students.
After holding tryouts, there are twelve Year 12 students and ten Year 11 students who are eligible to join the team.
Which of the following represents the number of ways the team can be selected?

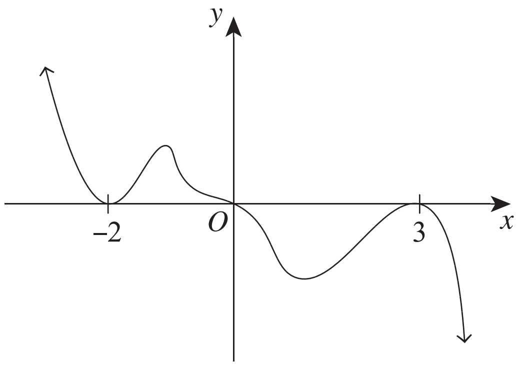
A. ${}^{22}C_{14}$

B. ${}^{22}P_{14}$

C. ${}^{12}C_8+{}^{10}C_6$

D. ${}^{12}C_8\times{}^{10}C_6$

5 Consider the graph.



Which of the following best represents the equation of the graph?

A. $y = \frac{1}{2}x^3(x+2)^2(x-3)^2$

B. $y = \frac{1}{2}x^3(x-2)^2(x-3)^2$

C. $y = -2x^3(x+2)^2(x-3)^2$

D. $y = -2x^3(x-2)^2(x-3)^2$

6 Given that α and β are both acute angles, evaluate $\cos(\alpha + \beta)$ if $\cos\alpha = \frac{12}{13}$ and $\cos\beta = \frac{4}{5}$.

A. $\frac{16}{65}$

B. $\frac{33}{65}$

C. $\frac{56}{65}$

D. $\frac{63}{65}$

7 A function is defined by $f(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 0$.
Which of the following represents the inverse function of $f(x)$?

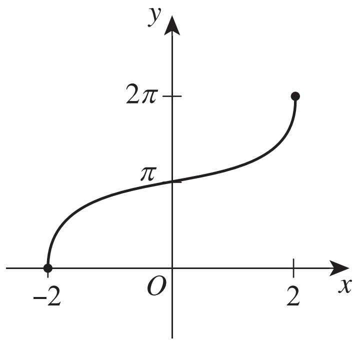
A. $f^{-1}(x) = -\sqrt{9 - x^2}$ for $0 \le x \le 3$

B. $f^{-1}(x) = \sqrt{9 - x^2}$ for $0 \le x \le 3$

C. $f^{-1}(x) = -\sqrt{9 - x^2}$ for $-3 \le x \le 0$

D. $f^{-1}(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 0$

8 Consider the graph.



Which of the following functions represents the graph?

A. $f(x) = 2\pi - 2\pi\cos^{-1} 2x$

B. $f(x) = 2\pi - 2\cos^{-1} 2x$

C. $f(x) = 2\pi - 2\pi\cos^{-1} \frac{1}{2}x$

D. $f(x) = 2\pi - 2\cos^{-1} \frac{1}{2}x$

9 The total surface area of a sphere is increasing at a constant rate of 20 cm^2 per second. At what rate per second is the volume of the sphere increasing when the radius of the sphere is 5 cm?

A. 8 cm^3

B. 10 cm^3

C. 50 cm^3

D. 2000π cm^3

10 Which of the following is the coefficient of x^4 in the expansion of $\left(x + \frac{2}{x}\right)\left(x^2 - 2x + 1\right)^4$?

A. –168

B. –42

C. 42

D. 168

SECTION II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{x^2+6}{x} \geq 5$. **3**

(b) Find the values of a and b if $\left(2-\sqrt{3}\right)^4 = a + b\sqrt{3}$. **2**

(c) The polynomial $P(x) = 2x^3 + x^2 + ax + 6$ has the roots α, $\dfrac{1}{\alpha}$ and β. **3**

Find the values of the THREE roots and, hence, find the value of a.

(d) It is given that $P(x) = x^4 - 5x^3 + ax^2 + bx - 48$, where a and b are constants, and that $x = 2$ is a double root of $P(x)$.

(i) Find the values of a and b. **2**

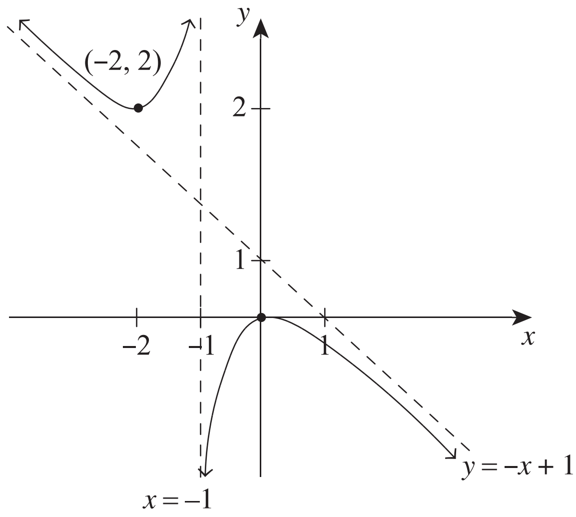
(ii) Factorise $P(x)$ completely into linear factors. **2**

(e) (i) Sketch the graph of $y = -x^2 + 2x + 3$. **2**

(ii) Hence, sketch the graph of $y = -|x|^2 + 2|x| + 3$. **1**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows the graph of $y = f(x)$.



(i) Sketch the graph of $y = \dfrac{1}{f(x)}$. **2**

(ii) Sketch the graph of $y = |f(-x)|$. **2**

(b) Consider the function $f(x) = \sin^{-1}(3 - 2x)$.

(i) State the domain and range. **2**

(ii) Hence, sketch the graph of $y = f(x)$. **1**

(c) Prove that $\cos\left(2\sin^{-1}\left(\dfrac{4}{5}\right)\right) = -\dfrac{7}{25}$. **2**

(d) Prove that $\dfrac{2\cos\theta + 1 + \cos 2\theta}{2\cos\theta - 1 - \cos 2\theta} = \cot^2\dfrac{\theta}{2}$. **3**

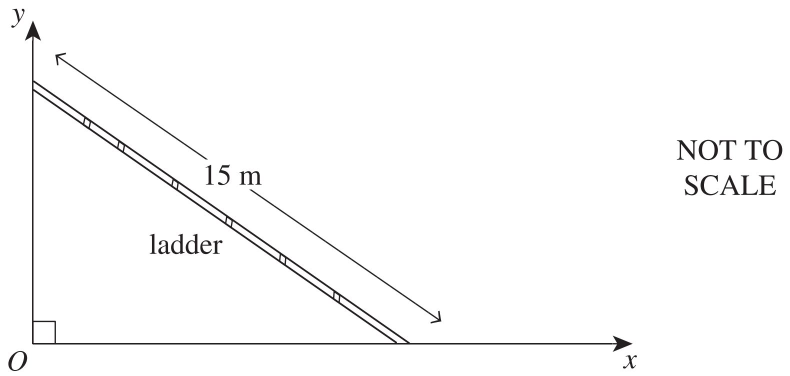
(e) (i) Calculate the number of permutations of the letters of the word TRIGONOMETRY. **1**

(ii) Calculate the number of permutations of the letters of the word TRIGONOMETRY where all the vowels are separated from each other. **2**

 TEN_Y11_MEx1_QA_2022

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) A ladder that is 15 metres in length is leaning against a vertical wall, as shown in the diagram. The foot of the ladder is sliding away from the wall at a rate of 2 m/s. **2**



Find the rate at which the top of the ladder is sliding down the wall when the foot of the ladder is 8 metres from the wall.

(b) Prove that ${}^{n}P_k - {}^{n-1}P_k = k \times {}^{n-1}P_{k-1}$. **2**

(c) In the expansion $(4+x)^n$, the coefficients of x^3 and x^4 are the same value. **2**
Find the value of n.

(d) There are 80 jelly beans of different colours contained in a bag. The distribution of the colours is shown in the table. **2**

Colour of jelly bean	pink	white	black	green	yellow	orange	brown	red
Number of jelly beans	2	3	4	10	12	15	16	18

What is the least number of jelly beans that can be selected from the bag to ensure that 8 of the selected jelly beans are the same colour?

Question 13 continues on page 8

Question 13 (continued)

(e) A chicken is roasted in an oven at 125°C. It is taken out of the oven and placed on a tray in a dining room in which the temperature is 25°C. The chicken cools down to 85°C after 8 minutes. The cooling rate is proportional to the difference between the chicken temperature and the room temperature according to the differential equation

$$\frac{dT}{dt} = -k(T - 25)$$

where k is a constant and T is the temperature of the chicken at any time, t, in minutes after the chicken has been placed in the dining room.

(i) Show that $T = 25 + Ae^{-kt}$ is a solution to the differential equation. **1**

(ii) Find the values of A and k. Give the value of k correct to four decimal places. **2**

(iii) How long will it take, in minutes, for the temperature of the chicken to reach 60°C? Give your answer correct to one decimal place. **1**

(f) Prove that $\sin 7x \cos 2x + \cos 6x \sin x = \sin 8x \cos x$. **3**

End of Question 13

TEN_Y11_MEx1_QA_2022

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the curve $f(x) = (2x - 1)^2 - 3$.

(i) State the largest negative domain so that $f(x)$ has an inverse function. **1**

(ii) Find the inverse function $f^{-1}(x)$. **2**

(iii) Find the intersection point where $f(x) = f^{-1}(x)$. **2**

(iv) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes. **1**

(b) (i) If $\cos(x + \theta) = k\cos(x - \theta)$, prove that $(1 + k)\tan x = (1 - k)\cot\theta$. **2**

(ii) Hence, solve $\cos(x + 30°) = 2\cos(x - 30°)$, $0 \le x \le 360°$. **2**

(c) The position of a particle moving along the x-axis is given by $x = 100e^{-0.2t} + 20t - 80$, where t is the time in seconds and x is the distance travelled in metres.

(i) Find the initial velocity and acceleration of the particle. **2**

(ii) When will the particle attain a velocity of half its limiting velocity? Give your answer correct to one decimal place. **2**

(iii) Show that the particle is always speeding up for any time (t, seconds). **1**

End of paper

MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

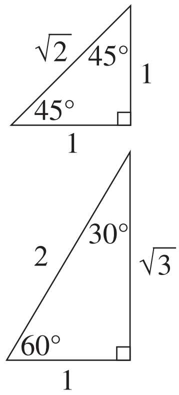
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

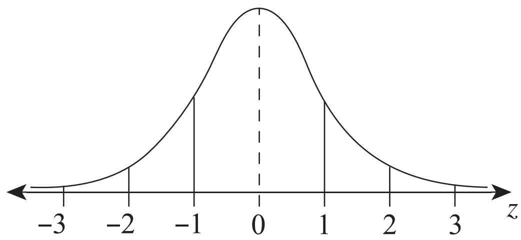
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \displaystyle\int_a^r f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$

$X \sim \text{Bin}(n, p)$

$\Rightarrow P(X = x)$

$= \dbinom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^r + \ldots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Neap

Final Examination 2022

NSW Year 11 Mathematics Extension 1

Section II Writing Booklet

Question Number

Student Name/Number: ______________________________

Instructions

Use a separate writing booklet for each question in Section II.

Write the number of this booklet and the total number of booklets that you have used for this question (e.g. $\boxed{1}$ of $\boxed{3}$) $\Rightarrow$ $\boxed{}$ of $\boxed{}$

this booklet

number of booklets for this question

Write in black pen.

You may ask for an extra writing booklet if you need more space.

If you have not attempted the question(s), you must still hand in a writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.

You may NOT take any writing booklets, used or unused, from the examination room.

Tick this box if you have continued this answer in another writing booklet. ☐

Neap

Final Examination 2022

NSW Year 11 Mathematics Extension 1

DIRECTIONS:

Write your name in the space provided.

Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.

Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one oval** per question.

A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct

A B C D

STUDENT NAME: ____________________

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I

MULTIPLE-CHOICE ANSWER SHEET

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D

STUDENTS SHOULD NOW CONTINUE WITH SECTION II