Neap

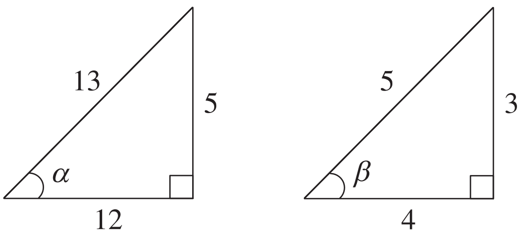
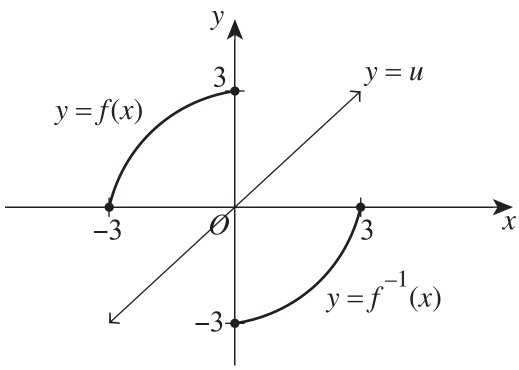
Final Examination 2022

NSW Year 11 Mathematics Extension 1

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **B** $\lvert 2x-1\rvert > 5$ $2x-1<-5$ $2x<-4$ $x<-2$ $2x-1>5$ $2x>6$ $x>3$ $\therefore x<-2,\ x>3$	ME–F1 Further Work with Functions ME11–2 Band E2
Question 2 **D** $4x^3-2x^2+8x-1=0$ $\alpha+\beta+\gamma=\frac{2}{4}$ $=\frac{1}{2}$ $\alpha\beta\gamma=\frac{1}{4}$ $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ $=\frac{\frac{1}{2}}{\frac{1}{4}}$ $=2$	ME–F2 Polynomials ME11–2 Band E2
Question 3 **C** $x=2\cos\theta-1$ $\cos\theta=\frac{x+1}{2}$ $y=4\sin\theta+1$ $\sin\theta=\frac{y-1}{4}$ $\left(\frac{x+1}{2}\right)^2+\left(\frac{y-1}{4}\right)^2=\cos^2\theta+\sin^2\theta$ $\frac{(x+1)^2}{4}+\frac{(y-1)^2}{16}=1$ $4(x+1)^2+(y-1)^2=16$	ME–F1 Further Work with Functions ME11–2 Band E2

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 4 **D** Selecting eight students from twelve total Year 12 students and six students from ten total Year 11 students is represented by ${}^{12}C_8 \times {}^{10}C_6$.	ME–A1 Working with Combinatorics ME11–5 Band E3
Question 5 **C** Since there are double roots at $x = -2$ and $x = 3$, either **A** or **C** is correct. As the graph shown is a negative polynomial of degree 7, the correct equation is $y = -2x^3(x+2)^2(x-3)^2$. Therefore, **C** is correct.	ME–F2 Polynomials ME11–2 Band E3
Question 6 **B** $\cos(\alpha + \beta)$ if $\cos\alpha = \frac{12}{13}$ and $\cos\beta = \frac{4}{5}$  $\sin\alpha = \frac{5}{13}$ $\sin\beta = \frac{3}{5}$ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $= \frac{12}{13} \times \frac{4}{5} - \frac{5}{13} \times \frac{3}{5}$ $= \frac{33}{65}$	ME–T2 Further Trigonometric Identities ME11–3 Band E2
Question 7 **A** $f(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 0$  Reading from the graph gives the inverse function as $f^{-1}(x) = -\sqrt{9 - x^2}$ for $0 \le x \le 3$.	ME–F1 Further Work with Functions ME11–2 Band E2

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 8 **D**	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3

Method 1:

Domain:

$-2 \le x \le 2$

$-1 \le \frac{1}{2}x \le 1$

Therefore, either **C** or **D** is correct.

Since $x = 0$:

$$y = 2\pi - 2\pi \cos^{-1}(0)$$
$$= 2\pi - 2\pi\left(\frac{\pi}{2}\right)$$
$$= 2\pi - \pi^2$$
$$\therefore\ y = 2\pi - 2\cos^{-1}\frac{1}{2}x$$

Therefore, **D** is correct.

Method 2:

Starting with $y = \cos^{-1} x$:

Increasing the horizontal dilation factor by 2 gives

$y = \cos^{-1}\left(\frac{x}{2}\right)$.

Increasing the vertical dilation factor by 2 gives

$y = 2\cos^{-1}\left(\frac{x}{2}\right)$.

Reflecting about the y-axis gives $y = 2\cos^{-1}\left(-\frac{x}{2}\right)$.

Recall that $\cos^{-1}(-x) = \pi - \cos^{-1} x$.

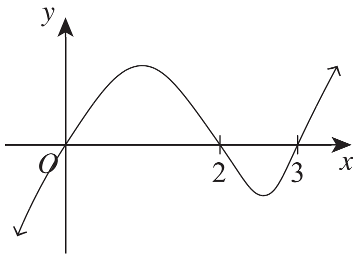
Hence, $\cos^{-1}\left(-\frac{x}{2}\right) = \pi - \cos^{-1}\left(\frac{x}{2}\right)$.

$$y = 2\cos^{-1}\left(-\frac{x}{2}\right)$$
$$= 2\left[\pi - \cos^{-1}\left(\frac{x}{2}\right)\right]$$
$$= 2\pi - 2\cos^{-1}\left(\frac{x}{2}\right)$$

Therefore, **D** is correct.

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **C** Given that $\frac{dA}{dt} = 20 \text{ cm}^2/\text{s}$: $\frac{dV}{dt} = \frac{dA}{dt} \times \frac{dV}{dA}$ $= \frac{dA}{dt} \times \frac{dV}{dr} \times \frac{dr}{dA}$ $V = \frac{4\pi r^3}{3}$ $\frac{dV}{dr} = 4\pi r^2$ $A = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$ $\frac{dV}{dt} = 20 \times 4\pi r^2 \times \frac{1}{8\pi r}$ $= 10r$ Since $r = 5$, $\frac{dV}{dt} = 50 \text{ cm}^3/\text{s}$.	ME–C1 Rates of Change ME11–4 Band E3
Question 10 **A** $\left(x + \frac{2}{x}\right)\left(x^2 - 2x + 1\right)^4$ $= \left(x + \frac{2}{x}\right)(x-1)^8$ $= \left(x + \frac{2}{x}\right)\left(x^8 - \binom{8}{1}x^7 + \binom{8}{2}x^6 - \binom{8}{3}x^5 + \binom{8}{4}x^4 - \binom{8}{5}x^3 + \ldots\right)$ coefficient of $x^4 = 1 \times -\binom{8}{5} + 2 \times -\binom{8}{3}$ $= -168$	ME–A1 Working with Combinatorics ME11–5 Band E4

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) $\left(\frac{x^2+6}{x} \geq 5\right) \times x^2,\ x \neq 0$ $x\left(x^2+6\right) \geq 5x^2$ $x^3-5x^2+6x \geq 0$ $x\left(x^2-5x+6\right) \geq 0$ $x(x-2)(x-3) \geq 0$ Sketching a graph of $y=x(x-2)(x-3)$ gives:  Reading from the graph gives the solutions for x as $0<x \leq 2,\ x \geq 3$ since $x \neq 0$.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct solution . . . . . .3 • Provides the solution but concludes $0 \leq x \leq 2,\ x \geq 3$ . . . . .2 • Shows some understanding of the problem 1
(b) $\left(2-\sqrt{3}\right)^4 = 2^4 - \binom{4}{1}\left(2^3\right)\left(\sqrt{3}\right) + \binom{4}{2}\left(2^2\right)\left(\sqrt{3}\right)^2 - \binom{4}{3}\left(2^1\right)\left(\sqrt{3}\right)^3 + \left(\sqrt{3}\right)^4$ $= 16-32\sqrt{3}+72-24\sqrt{3}+9$ $= 97-56\sqrt{3}$ $\therefore a=97,\ b=-56$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Provides the correct solution . . . . . .2 • Uses the binomial theorem OR equivalent merit. 1

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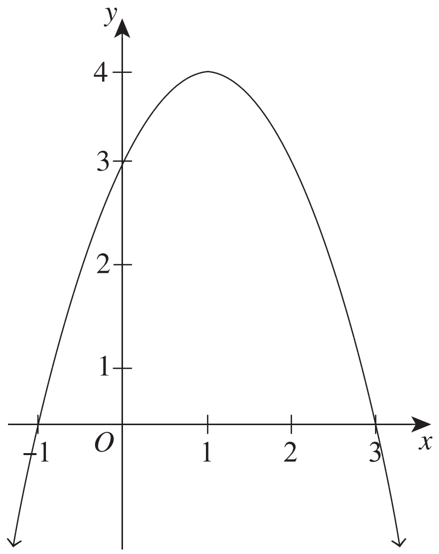
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $P(x)=2x^3+x^2+ax+6$ has the roots α, $\dfrac{1}{\alpha}$ and β. $\alpha\left(\dfrac{1}{\alpha}\right)(\beta)=-\dfrac{6}{2}$ $\beta=-3$ $\alpha+\dfrac{1}{\alpha}+\beta=-\dfrac{1}{2}$ $\alpha+\dfrac{1}{\alpha}-3=-\dfrac{1}{2}$ $2\alpha^2-5\alpha+2=0$ $(2\alpha-1)(\alpha-2)=0$ $\alpha=\dfrac{1}{2},\ \alpha=2$ Therefore, the roots are 2, $\dfrac{1}{2}$ and -3. $\left(\dfrac{1}{2}\right)(2)+\left(\dfrac{1}{2}\right)(-3)+(2)(-3)=\dfrac{a}{2}$ $a=-13$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Provides the correct solution . . . . . .3 • Obtains TWO correct roots OR equivalent merit. . . . . .2 • Obtains ONE correct root OR equivalent merit. 1
(d) (i) Since $x=2$ is a double root of $P(x)$: $P(2)=P'(2)$ $=0$ $P(x)=x^4-5x^3+ax^2+bx-48$ $P'(x)=4x^3-15x^2+2ax+b$ $P(2)=2^4-5(2^3)+a(2^2)+2b-48$ $=0$ $4a+2b=72$ $2a+b=36 \quad (1)$ $P'(2)=4(2^3)-15(2^2)+2a(2)+b$ $=0$ $4a+b=28 \quad (2)$ (2) – (1) gives: $4a+b-2a+b=28-36$ $2a=-8$ $a=-4$ Inserting $a=-4$ into (1) gives: $2(-4)+b=36$ $b=44$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Provides the correct solution . . . . . .2 • Obtains ONE correct equation OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Since $x = 2$ is a double root of $P(x)$, $(x-2)^2$ is a factor of $P(x)$. $P(x) = x^4 - 5x^3 - 4x^2 + 44x - 48$ $= (x^2 - 4x + 4)(x^2 - x - 12)$ (by inspection) $= (x-2)^2(x-4)(x+3)$ *Note: Consequential on answer to* ***Question 11(d)(i)***.	ME–F2 Polynomials ME11–2 Bands E2–E3 • Provides the correct solution . . . . . .2 • Obtains $(x-2)^2$ as a factor of $P(x)$ OR equivalent merit 1
(e) (i) $y = -x^2 + 2x + 3$ $= -(x+1)(x-3)$ x-intercepts (at $y = 0$): $0 = -(x+1)(x-3)$ $x = -1,\ 3$ y-intercept (at $x = 0$): $y = -(0+1)(0-3)$ $= 3$ Vertex: $x = \frac{-1+3}{2}$ $= 1$ $y = -(1)^2 + 2(1) + 3$ $= 4$ Therefore, the vertex is (1, 4). Sketching the graph gives: 	ME–F1 Further Work with Functions ME11–2 Band E2 • Provides the correct sketch . . . . . . .2 • Provides some correct features of the graph OR equivalent merit. 1

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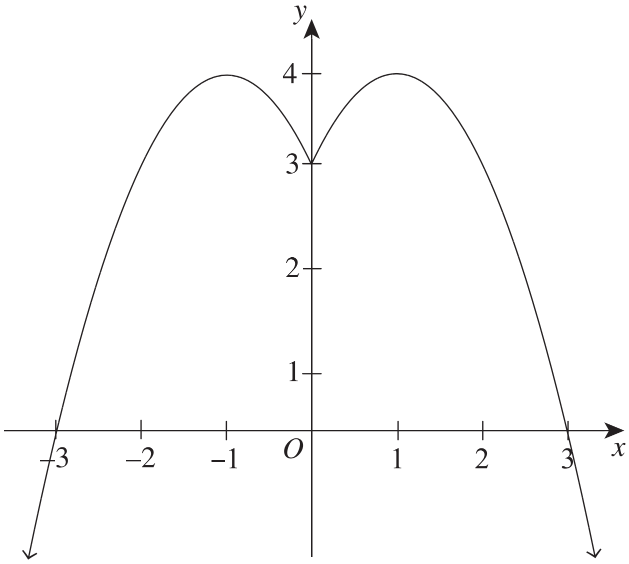
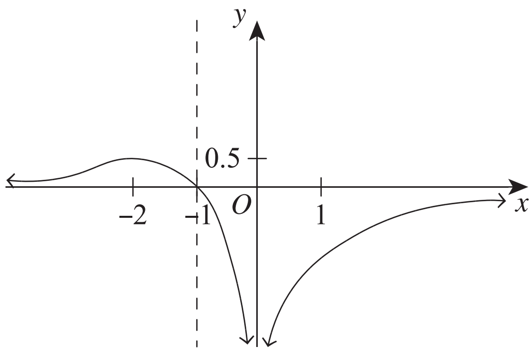
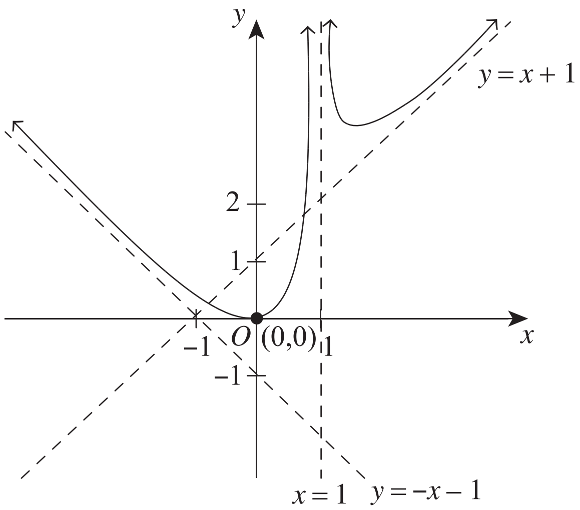
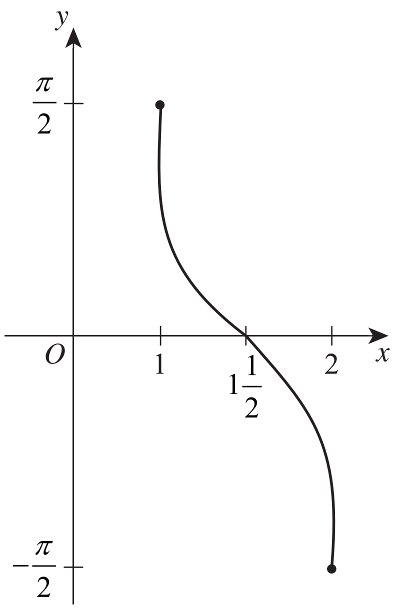
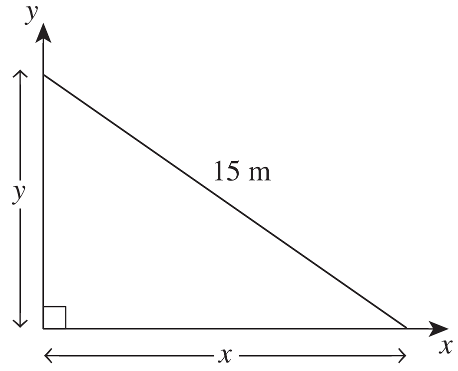
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Let $f(x) = -x^2 + 2x + 3$. $y = -\lvert x\rvert^2 + 2\lvert x\rvert + 3$ $= f(\lvert x\rvert)$ Sketching the graph gives: 	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct sketch 1
Question 12	
(a) (i) Sketching the graph of $y = \dfrac{1}{f(x)}$ gives: 	ME–F1 Further Work with Functions ME11–2 Band E2 • Provides the correct sketch . . . . . . .2 • Provides some correct features of the graph OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Sketching the graph of $y = \|f(-x)\|$ gives: 	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct sketch . . . . . . .2 • Provides some correct features of the graph OR equivalent merit. 1
(b) (i) $f(x) = \sin^{-1}(3-2x)$ Domain: $-1 \le 3-2x \le 1$ $-4 \le -2x \le -2$ $1 \le x \le 2$ Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	ME–T1 Inverse Trigonometric Functions ME11–3 Bands E2–E3 • Provides the correct solution . . . . . .2 • Provides the domain OR range 1
(ii) Sketching the graph of $y = f(x)$ gives: 	ME–T1 Inverse Trigonometric Functions ME11–3 E2 • Provides the correct sketch 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) Prove $\cos\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)=-\frac{7}{25}$. Let $\theta=\sin^{-1}\left(\frac{4}{5}\right)$. $\Rightarrow \sin\theta=\frac{4}{5}$ $\text{LHS}=\cos\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)$ $=1-2\sin^2\theta$ $=1-2\times\left(\frac{4}{5}\right)^2$ $=1-\frac{32}{25}$ $=-\frac{7}{25}$ $=\text{RHS}$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1
(d) $\text{LHS}=\frac{2\cos\theta+1+\cos2\theta}{2\cos\theta-1-\cos2\theta}$ $=\frac{2\cos\theta+1+2\cos^2\theta-1}{2\cos\theta-1-(2\cos^2\theta-1)}$ $=\frac{2\cos\theta+2\cos^2\theta}{2\cos\theta-2\cos^2\theta}$ $=\frac{2\cos\theta(1+\cos\theta)}{2\cos\theta(1-\cos\theta)}$ $=\frac{1+2\cos^2\frac{\theta}{2}-1}{1-\left(1-2\sin^2\frac{\theta}{2}\right)}$ $=\frac{\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}$ $=\cot^2\frac{\theta}{2}$ $=\text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Bands E3–E4 • Provides the correct solution . . . . . .3 • Makes substantial progress applying the double angle formulae involving $2\cos\theta$ . . . . . . .2 • Makes some progress applying the double angle formulae involving $\cos2\theta$ 1

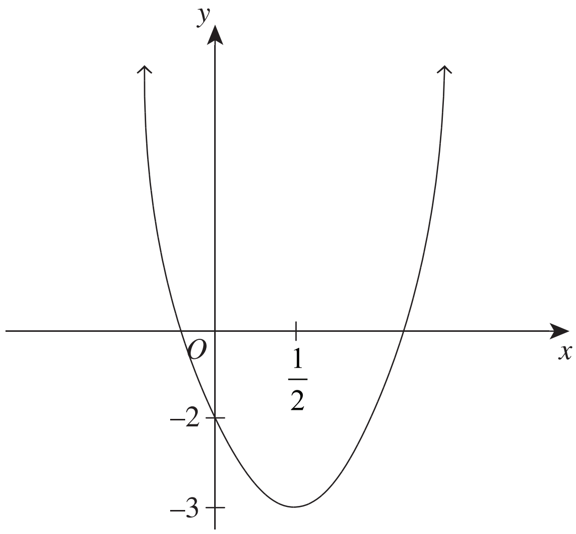
		Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e)	(i)	TRIGONOMETRY There are the letters T, T, R, R, G, M, N, Y, I, O, O and E. The repetitions are two of the letter T, two of the letter R and two of the letter O. There are 12 letters in total. $\text{number of permutations} = \dfrac{12!}{2!2!2!}$ $= \dfrac{12!}{8}$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Provides the correct solution 1
	(ii)	TRIGONOMETRY There are the letters T, T, R, R, G, M, N, Y, I, O, O and E. There are eight consonants and four vowels, and the repetitions are two of the letter T, two of the letter R and two of the letter O. Without considering the repetitions, the four vowels can occupy any **six positions** and the eight consonants can occupy eight positions. $\Rightarrow\ {}^6P_4$ for the vowels and $8!$ for the consonants. Hence, the total number of permutations by including the repetitions: $= \dfrac{{}^6P_4 \times 8!}{2!2!2!}$ $= {}^6P_4 \times 7!$	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Provides the correct solution . . . . . .2 • Provides the number of arrangements for the vowels OR consonants OR equivalent merit . . .1

<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>Question 13</td><td></td></tr>
<tr><td>(a) Let x and y be lengths as shown in the diagram.


$x^2 + y^2 = 15^2$
$y = \sqrt{225 - x^2}$
$\frac{dx}{dt} = 2$ m/s
$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$
$y = (225 - x^2)^{\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}(225 - x^2)^{-\frac{1}{2}} \times (-2x)$
$= -\frac{x}{\sqrt{225 - x^2}}$
$\frac{dy}{dt} = 2 \times -\frac{x}{\sqrt{225 - x^2}}$
$= -\frac{2x}{\sqrt{225 - x^2}}$
Since $x = 8$:
$\frac{dy}{dt} = -\frac{2(8)}{\sqrt{225 - 8^2}}$
$= -\frac{16}{\sqrt{161}}$ m/s
The rate at which the top of the ladder is sliding down the wall is $\frac{16}{\sqrt{161}}$ m/s.</td><td>ME–C1 Rates of Change
ME11–4 Bands E2–E3
• Provides the correct solution 2
• Correctly finds $\frac{dy}{dx}$ OR equivalent merit. . . . . .1</td></tr>
</table>

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $\text{LHS} = {}^{n}P_k - {}^{n-1}P_k$ $= \dfrac{n!}{(n-k)!} - \dfrac{(n-1)!}{(n-1-k)!}$ $= \dfrac{n! - (n-k)(n-1)!}{(n-k)!}$ $= \dfrac{(n-1)!\left[n-(n-k)\right]}{(n-k)!}$ $= \dfrac{(n-1)! \times k}{(n-k)!}$ $= k \times \dfrac{(n-1)!}{(n-1-(k-1))!}$ $= k \times {}^{n-1}P_{k-1}$ $= \text{RHS}$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1
(c) $(4+x)^n = 4^n + \binom{n}{1}4^{n-1}x + \binom{n}{2}4^{n-2}x^2 + \binom{n}{3}4^{n-3}x^3 + \binom{n}{4}4^{n-4}x^4 + \ldots$ coefficient of x^3 = coefficient of x^4 $\binom{n}{3}4^{n-3} = \binom{n}{4}4^{n-4}$ $\dfrac{n!}{(n-3)!3!} \times 4^{n-3} = \dfrac{n!}{(n-4)!4!} \times 4^{n-4}$ $\dfrac{1}{(n-3)(n-4)!3!} \times 4 = \dfrac{1}{(n-4)!4 \times 3!}$ $n-3 = 16$ $n = 19$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1
(d) From 2 pink, 3 white, 4 black, 10 green, 12 yellow, 15 orange, 16 brown and 18 red jelly beans, selecting a maximum of 7 from each colour gives: 2 pink, 3 white, 4 black, 7 green, 7 yellow, 7 orange, 7 brown and 7 red $\text{total} = 2+3+4+7+7+7+7+7$ $= 44$ Using the pigeonhole principle: 44 + 1 = 45. Hence, the least number of jelly beans that can be selected is 45 to ensure that 8 of the selected jelly beans are the same colour.	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) (i) $T=25+Ae^{-kt}$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k\left(T-25\right)$, since $T=25+Ae^{-kt}$	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution 1
(ii) $T=25+Ae^{-kt}$ $t=0$, $T=125$ $125=25+Ae^{0}$ $A=100$ $T=25+100e^{-kt}$ $t=8$, $T=85$ $85=25+100e^{-8k}$ $100e^{-8k}=60$ $e^{8k}=\frac{100}{60}=\frac{5}{3}$ $8k=\ln\left(\frac{5}{3}\right)$ $k=\frac{1}{8}\ln\left(\frac{5}{3}\right)$ $=0.0639$ (correct to 4 decimal places)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution . . . . . . 2 • Finds A OR equivalent merit 1
(iii) $T=25+100e^{-0.0639t}$ $60=25+100e^{-0.0639t}$ $e^{-0.0639t}=\frac{35}{100}$ $=\frac{7}{20}$ $t=\frac{\ln\left(\frac{7}{20}\right)}{-0.0639}$ $=16.4$ min (correct to 1 decimal place) *Note: Consequential on answer to **Question 13(e)(ii)**.*	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(f) $\text{LHS} = \sin 7x\cos 2x + \cos 6x \sin x$ $= \frac{1}{2}\left(\sin(7x+2x)+\sin(7x-2x)\right)$ $+\frac{1}{2}\left(\sin(6x+x)-\sin(6x-x)\right)$ $= \frac{1}{2}(\sin 9x + \sin 5x) + \frac{1}{2}(\sin 7x - \sin 5x)$ $= \frac{1}{2}(\sin 9x + \sin 7x)$ $= \frac{1}{2}\left(\sin(8x+x)+\sin(8x-x)\right)$ $= \frac{1}{2}(2\sin 8x\cos x)$ (since $2\sin A\cos B$) $= \sin(A+B)+\sin(A-B)$) $= \sin 8x \cos x$ $= \text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Bands E3–E4 • Provides the correct solution . . . . . .3 • Makes substantial progress applying the product-to-sum formulae and the sum-to-product formula. . . . . .2 • Makes some progress applying the product-to-sum formulae 1
Question 14	
(a) (i) Sketching the curve of $f(x) = (2x-1)^2 - 3$ gives:  Reading from the graph: domain of $f(x): x \le \frac{1}{2}$, so that $f^{-1}(x)$ exists	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct solution 1

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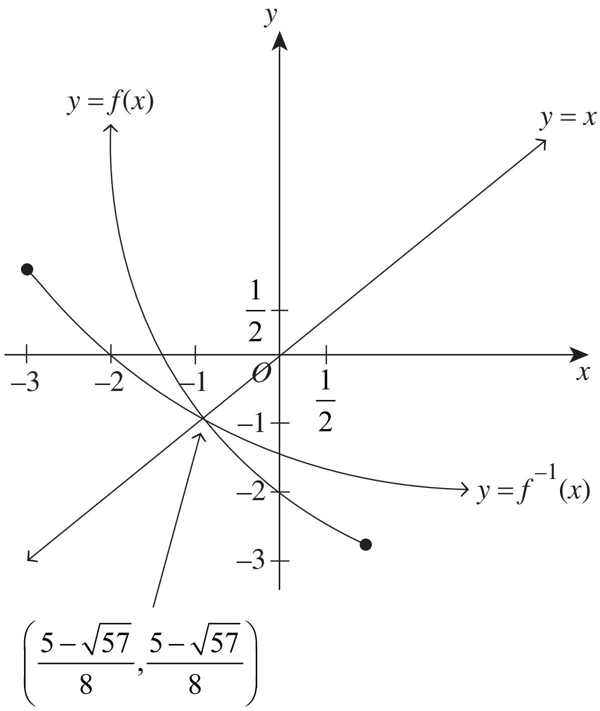
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) domain $f(x): x \le \frac{1}{2} \Rightarrow$ range $f^{-1}(x): y \le \frac{1}{2}$ range $f(x): y \ge -3 \Rightarrow$ domain $f^{-1}(x): x \ge -3$ $y = (2x-1)^2 - 3$ Interchanging x and y: $x = (2y-1)^2 - 3$ $(2y-1)^2 = x+3$ $2y - 1 = \pm\sqrt{x+3}$ $y = \frac{1 \pm \sqrt{x+3}}{2}$ $\therefore f^{-1}(x) = \frac{1-\sqrt{x+3}}{2}$, since range $f^{-1}(x): y \le \frac{1}{2}$ *Note: Consequential on answer to* ***Question 14(a)(i)***.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1
(iii) $f(x) = f^{-1}(x)$ $= x$ $(2x-1)^2 - 3 = x$ $4x^2 - 4x + 1 - 3 = x$ $4x^2 - 5x - 2 = 0$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(4)(-2)}}{2(4)}$ $= \frac{5 \pm \sqrt{57}}{8}$ $\therefore x = \frac{5-\sqrt{57}}{8}$, since $x \le \frac{1}{2}$ The point of intersection is $\left(\frac{5-\sqrt{57}}{8}, \frac{5-\sqrt{57}}{8}\right)$. *Note: Consequential on answer to* ***Question 14(a)(i)***.	ME–F1 Further Work with Functions ME11–2 Band E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(iv)  *Note: Consequential on answers to* ***Questions 14(a)(i), (ii) and (iii)***.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Provides the correct sketch 1
(b) (i) $\cos(x+\theta)=k\cos(x-\theta)$ $\cos x\cos\theta-\sin x\sin\theta=k\left(\cos x\cos\theta+\sin x\sin\theta\right)$ $\left(1-k\right)\cos x\cos\theta=\left(k+1\right)\sin x\sin\theta$ $\frac{1-k}{k+1}=\frac{\sin x\sin\theta}{\cos x\cos\theta}$ $=\tan x\tan\theta$ $\left(1-k\right)\cot\theta=\left(k+1\right)\tan x$	ME–T2 Further Trigonometric Identities ME11–3 Band E3 • Provides the correct solution . . . . . .2 • Shows some understanding of the problem 1
(ii) $\cos(x+30°)=2\cos(x-30°)$ $3\tan x=-\cot 30°\quad$ (from part (i)) $3\tan x=-\sqrt{3}$ $\tan x=-\frac{\sqrt{3}}{3}$ $\therefore x=150°,\ 330°,$ since $0\le x\le 360°$ *Note: Consequential on answer to* ***Question 14(b)(i)***.	ME–T2 Further Trigonometric Identities ME11–3 Bands E2–E3 • Provides the correct solution . . . . . .2 • Provides ONE correct answer OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (i) $x = 100e^{-0.2t} + 20t - 80$ $v = \frac{dx}{dt}$ $= -20e^{-0.2t} + 20$ $a = \frac{dv}{dt}$ $= 4e^{-0.2t}$ When $t = 0$: $v = -20e^{0} + 20$ $= 0$ m/s When $t = 0$: $a = 4e^{0}$ $= 4$ m/s^2	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution . . . . . .2 • Provides velocity OR acceleration. OR • Equivalent merit 1
(ii) $v = 20 - 20e^{-0.2t}$ $t \to \infty,\ e^{-0.2t} \to 0$ $v \to 20$ $\therefore v = 20$ m/s When $v = 10$ m/s, $t = ?$ $20 - 20e^{-0.2t} = 10$ $20e^{-0.2t} = 10$ $e^{-0.2t} = \frac{10}{20}$ $= \frac{1}{2}$ $t = \frac{\ln\left(\frac{1}{2}\right)}{-0.2}$ $= 3.5$ s (correct to 1 decimal place)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Provides the correct solution . . . . . .2 • Finds $v = 20$ m/s AND makes substantial progress finding t when $v = 10$ m/s. 1
(iii) $a = 4e^{-0.2t}$ Since $e^{-0.2t} > 0,\ a = 4e^{-0.2t} > 0$. $v = 20\left(1 - e^{-0.2t}\right)$ For $t > 0,\ e^{-0.2t} < 1$. $\therefore 1 - e^{-0.2t} > 0$ $\therefore v = 20\left(1 - e^{-0.2t}\right) > 0$ Since $v > 0$ and $a > 0$, the particle is always speeding up.	ME–C1 Rates of Change ME11–4 Band E3 • Provides the correct solution 1