CSSA

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAMINATION ROOM

Centre Number

Student Number

Mathematics Advanced

Morning Session
Monday, 8 August 2022

General Instructions

- Reading time – 10 mins
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Use the Multiple-Choice Answer Sheet provided for Section I
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I - 10 marks (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 34)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

Section I
10 marks

Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1 – 10.

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1 The table below shows the number of students that make up a school's debating team.

	Junior	Senior
Male	4	6
Female	10	5

A student is picked at random from the debating team. Given that the student is female, what is the probability the student is a junior?

A. $\frac{2}{3}$

B. $\frac{3}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{3}$

2 Which of the following is equivalent to $\frac{2^n - 2^{n-1}}{2^{n+1} + 2^n}$?

A. $-\frac{1}{4}$

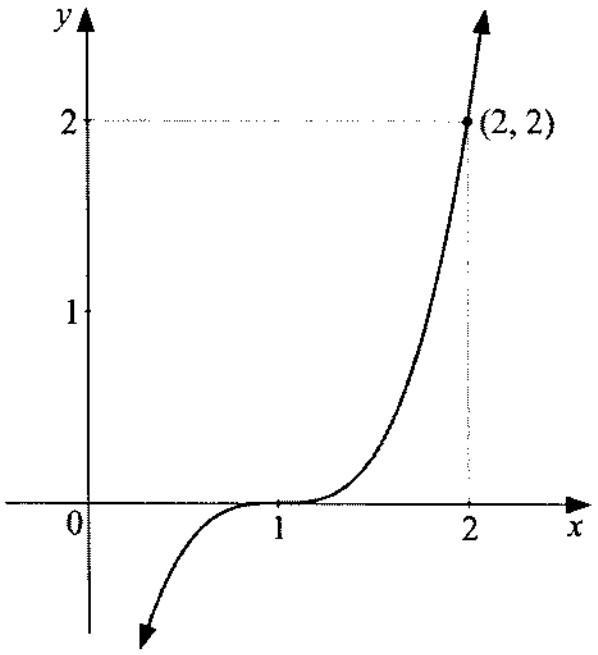
B. $-\frac{1}{2}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

3 The graph of the function $f(x) = x^3$ is transformed in such a way that the graph of the transformed function is given by $g(x) = kf(x + b)$.

The graph of $y = g(x)$ passes through the point $(2, 2)$ as shown in the diagram below.



What are the values of k and b?

A. $k = -1$ and $b = 2$

B. $k = 1$ and $b = 2$

C. $k = 2$ and $b = 1$

D. $k = 2$ and $b = -1$

4 Which description best fits the correlation of the following bivariate data?

x	12	14	15	17	22
y	−92	−85	−79	−80	−72

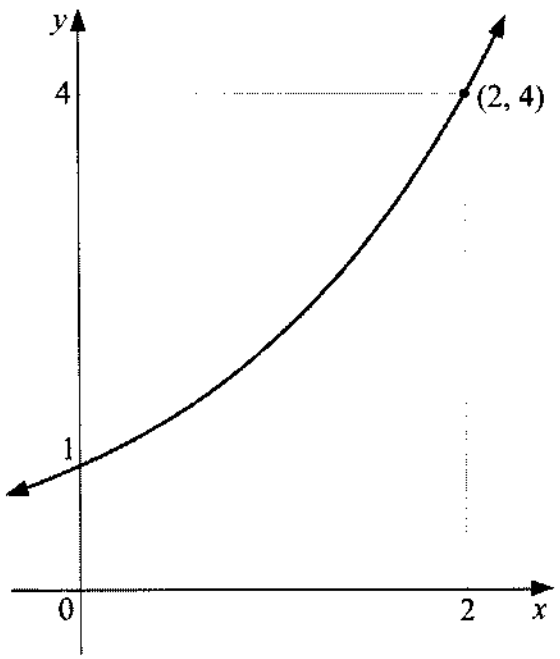
A. Weak, negative correlation

B. Strong, positive correlation

C. Strong, negative correlation

D. Moderate, positive correlation

5 The graph of the function $y = 2^x$ is shown below.



What is the area bounded by the curve $y = 2^x$ and the y-axis from $y = 1$ to $y = 4$?

A. $\frac{3}{\ln 2}$

B. $8 - \frac{3}{\ln 2}$

C. $3 \ln 2$

D. $4 - 3 \ln 2$

6 Consider the function $f(x) = k\cos\left(6x + \frac{\pi}{2}\right) + c$, where c and k are constants.

The function has a minimum value of -1 and a maximum value of 5.

Which row of the table shows the correct amplitude, period and phase?

	Amplitude	Period	Phase
A.	5	$\frac{\pi}{3}$	$\frac{\pi}{12}$
B.	3	$\frac{\pi}{3}$	$\frac{\pi}{12}$
C.	3	$\frac{\pi}{3}$	$\frac{\pi}{2}$
D.	3	$\frac{3}{\pi}$	$\frac{\pi}{12}$

7 Let $\log_a w = 3, \log_a x = 5$ and $\log_a y = 7$ where $a > 0$.

What is the value of $\log_a\left(\frac{w^2y}{x}\right)$?

A. 2.6

B. 5

C. 8

D. 12.6

8 A bricklayer is building a wall in the shape of a triangle where each row contains 3 bricks less than the row immediately below.

The bottom row of the wall contains 150 bricks.

If it takes the bricklayer 10 seconds to install each brick, how many rows of the wall will be completed after 6 hours?

A. 16

B. 17

C. 18

D. 19

9 For what values of b is the function $y = x^4 + bx^3 - 3x^2$ concave up and decreasing at the point where $x = 1$?

A. $\left(-1, \frac{2}{3}\right)$

B. $[-1, \infty)$

C. $\left(-\infty, \frac{2}{3}\right)$

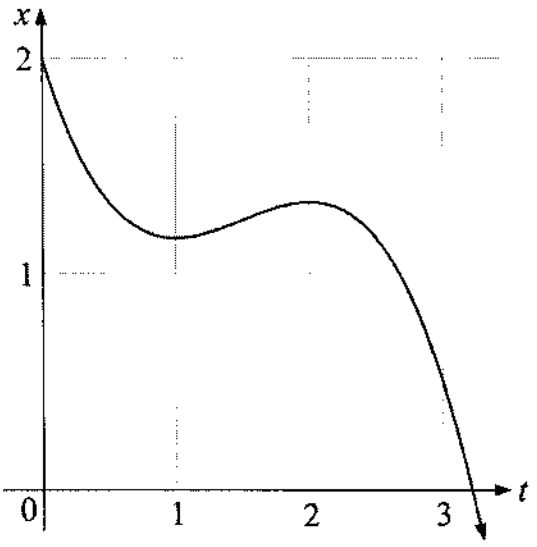
D. $(-\infty, -1]$

10 A particle P moves along a straight line such that its acceleration with respect to time is given by $a = -2t + 3$, where a is the acceleration of the particle at time t seconds for $t \geq 0$.

The particle is at rest when $t = 1$ and $t = 2$.

Which of the following graphs best represents the displacement of the particle?

A.



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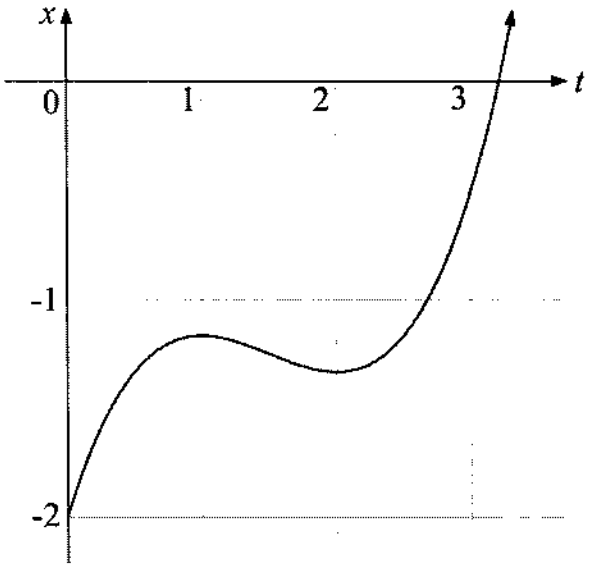

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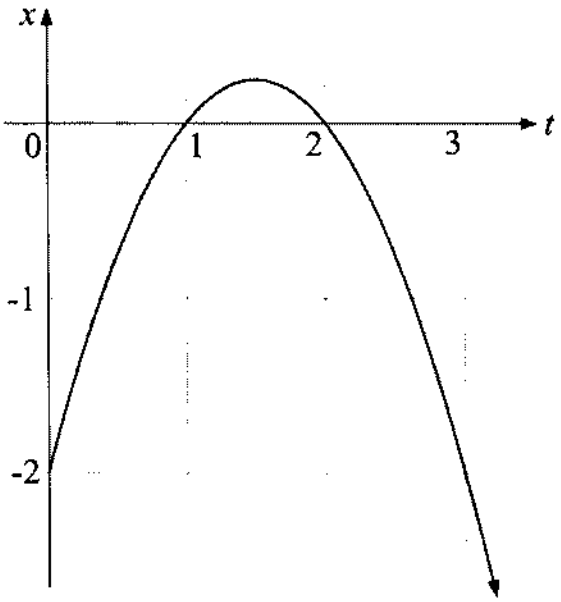
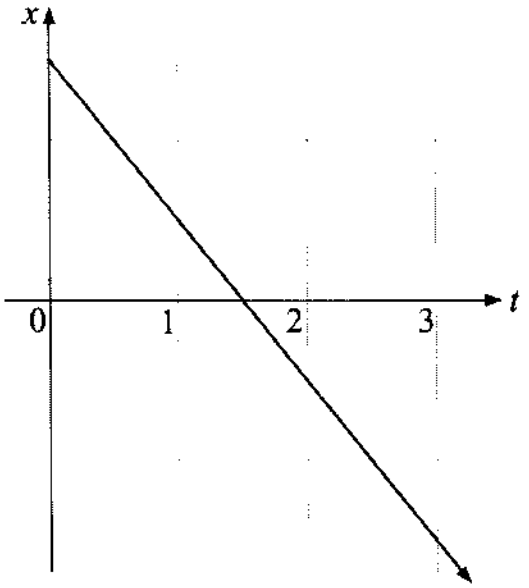


D.



Section II
90 marks

Attempt Questions 11 – 35
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 35. If you use this space, clearly indicate which question you are answering.
- Extra writing booklets are available.

Section II begins on page 9

Question 11 (2 marks)

Solve $|4x + 1| = 9$. **2**

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Question 12 (3 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = 4 - x^2$. **3**

By writing down an expression for the function $h(x) = f\big(g(x)\big)$, find the first derivative of the function $y = h(x)$.

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Question 13 (3 marks)

The letters of the word ALGEBRA are written on seven separate cards.

A	L	G	E	B	R	A

Two cards are chosen at random without replacement.

(a) What is the probability that the first card chosen is the letter A? **1**

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(b) What is the probability that only one of the cards chosen is the letter A? **2**

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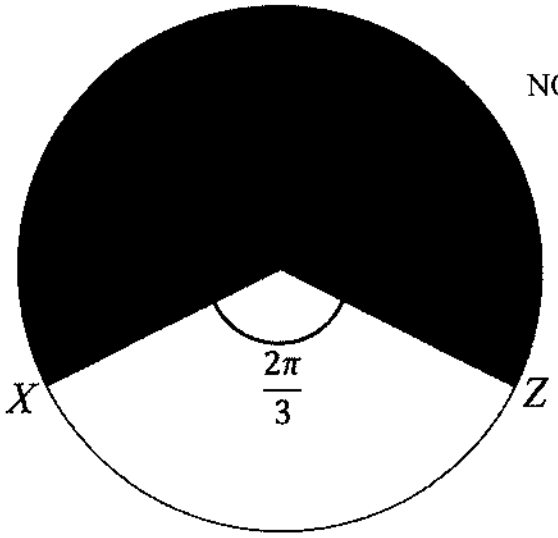
Question 14 (3 marks)

The circle shown has centre O and the angle of minor sector XOZ is $\frac{2\pi}{3}$ radians. **3**



Find the perimeter of the shaded region, given that the area of minor sector XOZ is 12π cm^2.

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Question 15 (3 marks)

Katrina runs a business refurbishing old computers. **3**

Source: Michael Zaouk, 2022 CSSA Trial HSC Examinations Committee – Mathematics Advanced. Used with permission.

The cost, C dollars, of refurbishing n computers is given by $C = 3n^2 + 100$.

The income, I dollars, received from selling n computers is given by $I = 80n$.

Determine the minimum and maximum number of computers that Katrina can refurbish and sell to make a profit.

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Question 16 (4 marks)

A car salesperson records the age in years and the price in dollars of six cars that are for sale.

Age (A)	3	5	5	6	8	10
Price (P)	23 000	18 000	16 500	12 000	9700	7000

(a) Find the equation of the least-squares regression line, giving each value to the nearest whole number, and use the equation to estimate the cost of a car that is 4 years old. **3**

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(b) John has a rare classic car that is now 50 years old. Explain why the least-squares regression line in part (a) cannot be used to determine the value of John's car. **1**

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Question 17 (3 marks)

The following table shows the probability distribution of a discrete random variable X. **3**

X	0	1	2	3	4
$P(X = x)$	0.05	p	p	q	0.01

Given that $E(X) = 2.08$, find the values of p and q.

Question 18 (3 marks)

The shoe size of women in a town is normally distributed with a mean size of 8 and a standard deviation of 1. **3**

A shoe shop in the town sells women's shoes ranging from size 5 to size 10.

Given that there are 4000 women in the town, how many would be expected to find shoes that fit them in this shop?

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Question 19 (4 marks)

Last month eight houses were sold in Oldsville.

The selling prices of the houses were:

$480 000 $505 000 $517 000 $528 000 $528 000 $552 000 $580 000 $980 000

(a) Show that the value of the most expensive house sold is an outlier for this data set. **2**

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(b) Explain the effect of the outlier on the mean and median selling prices for this data set. Justify your answer with relevant calculations. **2**

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Question 20 (3 marks)

Consider the function $f(x) = 2x - \sin x$.

(a) Show that $f(x)$ is an odd function. **1**

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(b) Hence, or otherwise, find the area bounded by the curve $y = f(x)$ and the x-axis, from $x = -\pi$ to $x = \pi$, leaving your answer in exact form. **2**

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Question 21 (2 marks)

Determine the values of x for which the graph of $y = 3\cos\left(\frac{5x-1}{2}\right)$ crosses the x-axis in the interval $0 \leq x \leq \pi$. **2**

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Question 22 (5 marks)

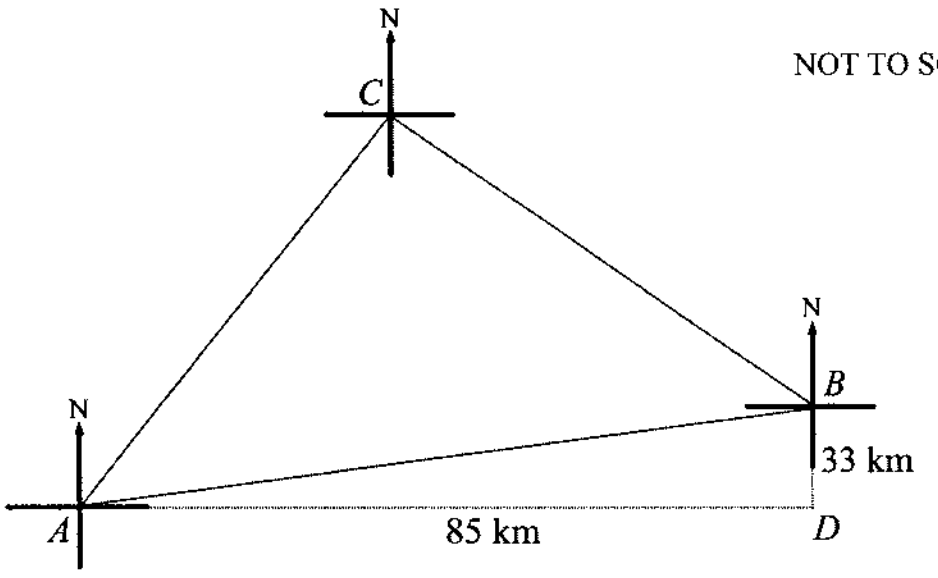
Ambarella (A), Bilberry (B) and Calabash (C) are three country towns. **5**

Bilberry is 85 km east and 33 km north of Ambarella.

Calabash is on a bearing of 026° from Ambarella and 321° from Bilberry.

Jo completes a cross-country drive which starts and finishes in Ambarella and passes through Bilberry and Calabash.

Calculate the total distance that Jo travelled, correct to the nearest kilometre.



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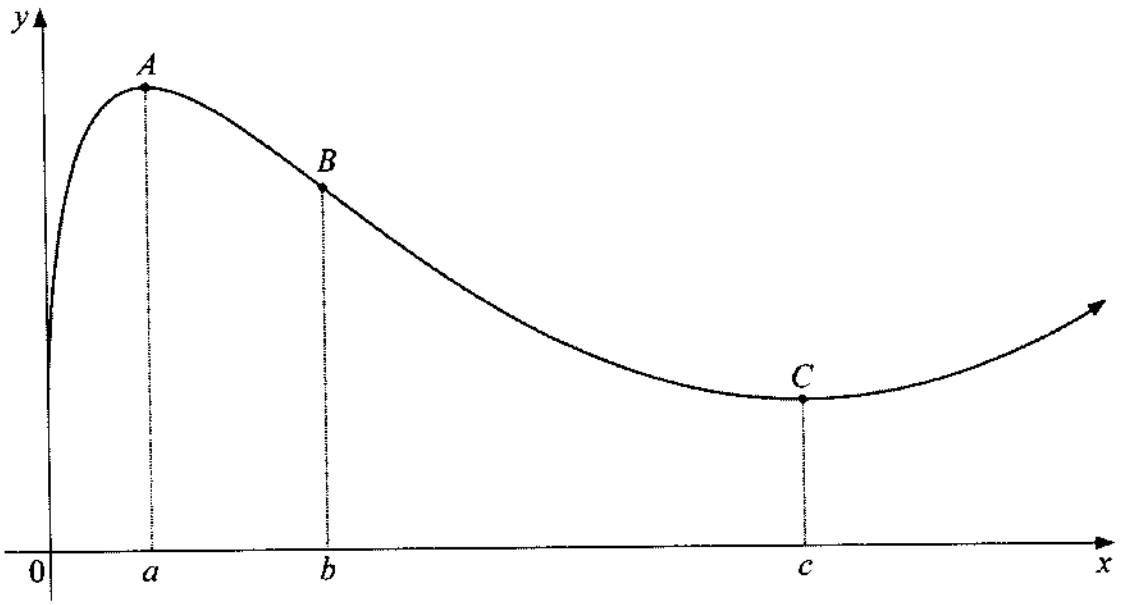
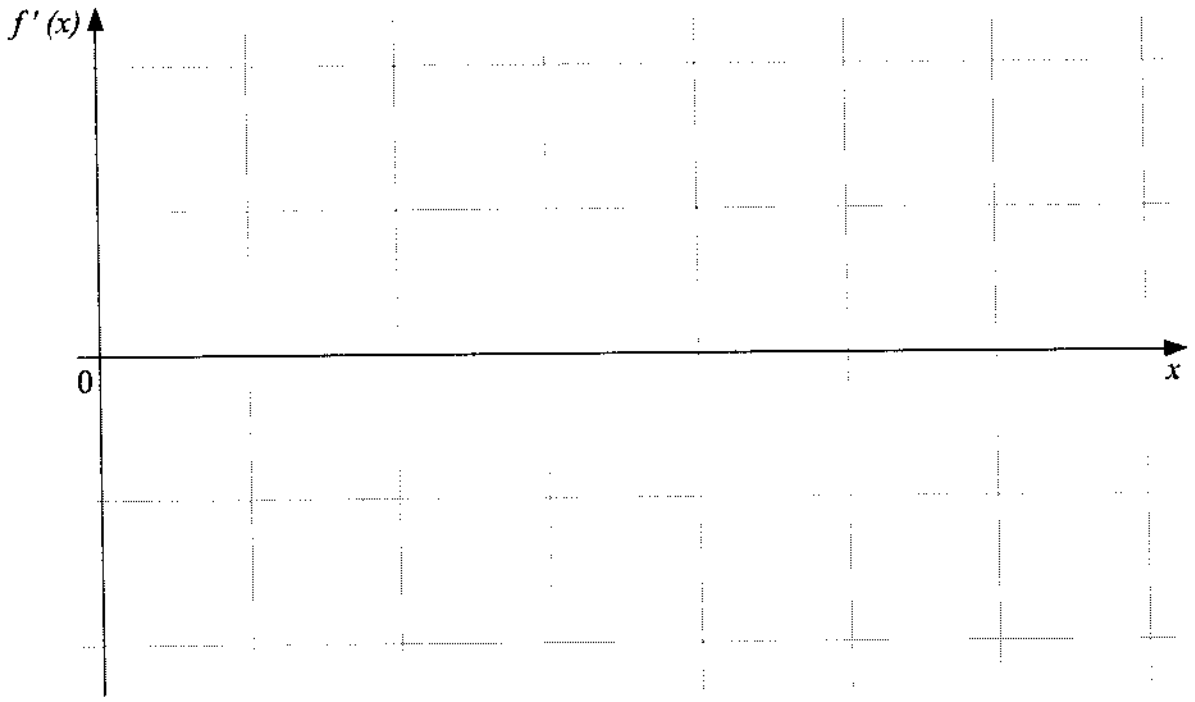
Question 23 (2 marks)

The following diagram shows the graph of the function $y = f(x)$. **2**



The function has a local maximum turning point at A where $x = a$, a point of inflection at B where $x = b$ and a local minimum turning point at C where $x = c$.

On the grid below, sketch the graph of the first derivative, $y = f'(x)$.



Question 24 (4 marks)

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sec^2\left(\frac{x}{3}\right), & \text{for } 0 \le x \le \pi \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k = \frac{1}{3\sqrt{3}}$. **2**

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(b) Find the median of X, correct to two decimal places. **2**

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Question 25 (3 marks)

Consider the function $f(x) = \dfrac{4x}{\sqrt{x-2}}$. **3**

Find any stationary points and determine their nature.

Question 26 (3 marks)

Let $f(x) = x^2 - 2x$. **3**

Sketch the graph of $y = 2f(1 - x) - 6$, showing the location of the vertex.

Question 27 (3 marks)

The rate of fuel consumption of a particular vehicle may be modelled by the formula **3**

$$R = 5 + 4e^{-0.5t},$$

where R is the rate of fuel consumption, in litres per hour, after t hours of travel.

In the first hour of travel, the vehicle consumed 25 litres of fuel.

Determine the amount of fuel consumed after 4 hours of travel, correct to the nearest litre.

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Question 28 (7 marks)

Maroun deposits $500 000 into a retirement fund which earns compound interest at the rate of 0.3% per month.

Maroun withdraws spending money from the retirement fund at the end of each month.

The incomplete table below shows the account activity for the first four months and the balance at the end of the fifth month.

Month	Principal (P)	Interest (I)	Withdrawal (M)	Balance ($P + I - M$)
1	500 000.00	1500.00	2000	499 500.00
2	499 500.00	1498.50	2500	498 498.50
3	498 498.50	1495.50	3000	496 994.00
4	496 994.00	1490.98	3800	
5				491 969.03

(a) Calculate the amount of money that Maroun withdrew at the end of the fifth month. **3**

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Question 28 continues on page 25

Question 28 (continued)

(b) Maroun is going to withdraw \$7500 at the end of each month, starting from the end of the sixth month. **2**

The recurrence relation $A_n = A_{n-1} \times 1.003 - 7500$ models this situation, where A_n is the balance in the retirement fund at the end of the nth month.

Determine the balance in Maroun's retirement fund at the end of the seventh month.

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(c) Maroun is offered an alternative retirement fund, where interest is paid at 3.2% per annum, compounded daily. **2**

By calculating the effective annual rate of interest for both retirement funds, or otherwise, determine whether Maroun should remain with the current retirement fund or change to the alternative retirement fund.

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End of Question 28

Question 29 (4 marks)

A rocket is returning back to Earth after completing a space mission.

Its distance from Earth can be modelled by the equation

$$S = 55000 \times 3^{-0.182t},$$

where S is the distance from Earth in kilometres at time t hours after it begins its return journey.

(a) Find the speed of the rocket after 2 hours on its return journey. **2**

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(b) When the rocket reaches a distance of 1000 km from Earth, a parachute is released. **2**

How long, in hours, after the rocket begins its return journey does it release its parachute? Give your answer correct to two decimal places.

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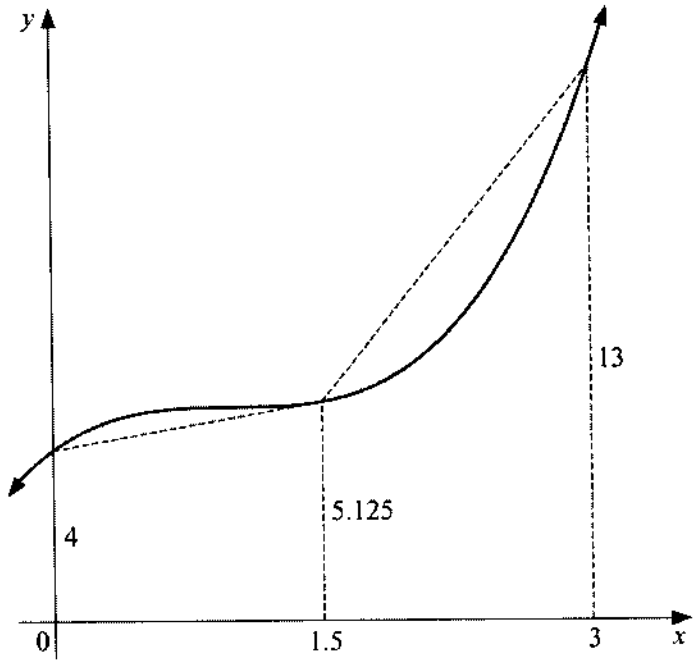
Question 30 (3 marks)

The graph below shows the curve $y = x^3 - 3x^2 + 3x + 4$. **3**

Using two applications of the Trapezoidal rule from $x = 0$ to $x = 3$, determine how much the Trapezoidal rule overestimates the exact area under the curve.



Question 31 (5 marks)

A ball is dropped from a height H metres above the ground. Each time it strikes the ground, it rebounds to a lower height which is $q\%$ of its previous height.

Let h_n be the height reached by the ball after the nth bounce.

(a) Show that, after the second bounce, the ball rebounds to a height given by **2**

$$h_2 = \frac{q^2 H}{(100)^2}.$$

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(b) Show that the total distance, D, travelled by the ball is given by **3**

$$D = \frac{H(100 + q)}{100 - q}.$$

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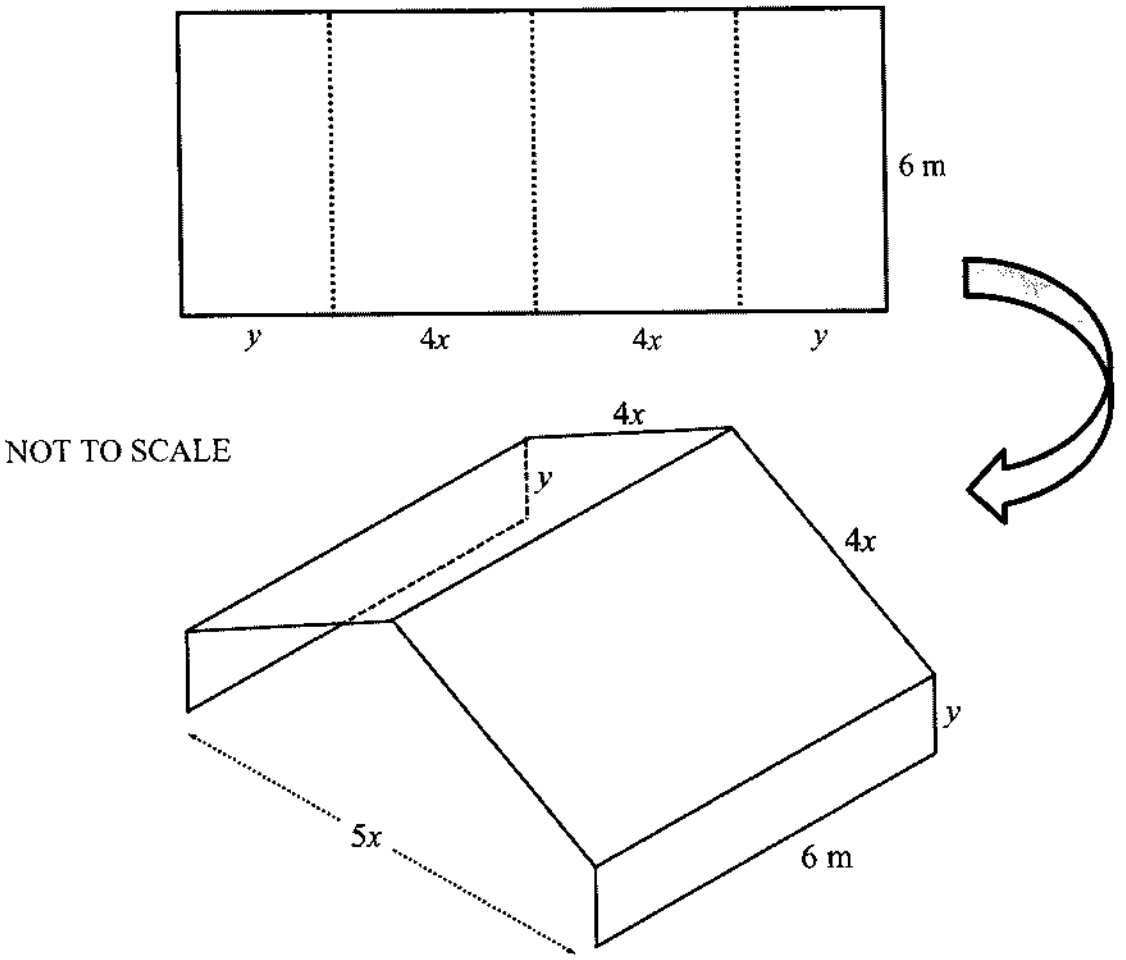
Question 32 (8 marks)

A rectangular canvas sheet is folded to form the cover of a tent with a pentagonal cross-section, as shown in the following diagrams.



The vertical heights on the sides of the tent are y metres and the slant lengths are $4x$ metres. The horizontal distance between the vertical sides is $5x$ metres and the width of the tent is 6 metres.

(a) Show that $y = 10 - 4x$, given the area of the canvas sheet is 120 m^2. **1**

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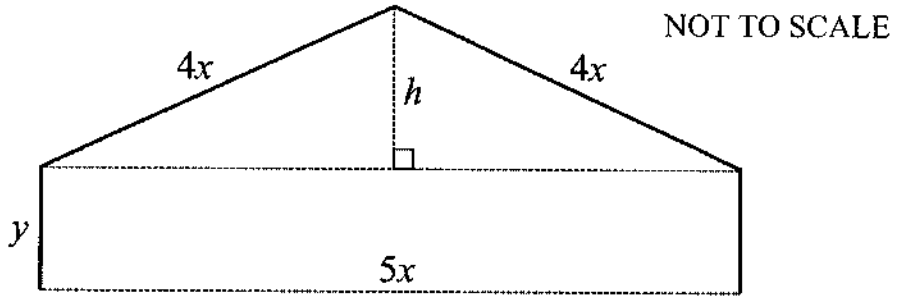
Question 32 continues on page 30

Question 32 (continued)

The following diagram shows the cross-section of the tent, where h is the height of the top of the tent from the vertical sides.



(b) Show that $h = \frac{\sqrt{39}x}{2}$. **2**

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Question 32 continues on page 31

Question 32 (continued)

(c) Find the value of x which maximises the volume of the tent. **3**

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(d) Hence, find the maximum volume of the tent, correct to one decimal place. **2**

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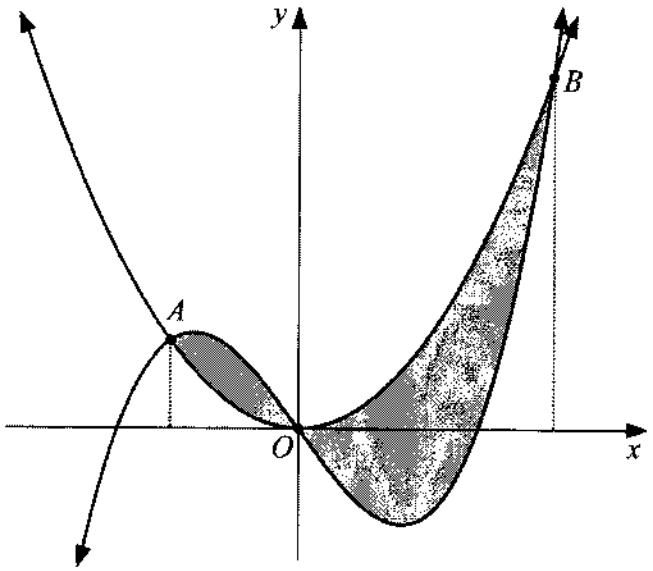
End of Question 32

Question 33 (4 marks)

The diagram shows the graphs of $y = x^3 - 2x$ and $y = x^2$. **4**



The graphs intersect at A, B and the origin, O.

By finding the x-coordinates of A and B, show that the area of the shaded region is $\frac{37}{12}$.

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Question 34 (3 marks)

Prove that $\frac{\cos\theta}{\cos(90° - \theta)}(1 + \tan^2\theta) + \frac{\sin\theta}{\cos\theta}[1 + \tan^2(90° - \theta)] = 2\text{cosec}\theta\sec\theta.$ **3**

Question 35 (3 marks)

It is known that $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1 - \ln x}{x^2}$. (Do NOT prove this.) **3**

Find $\int \frac{1 - \ln x}{x \ln x} dx$.

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End of Examination

Section II extra writing space

If you use this space, clearly indicate which question you are answering by writing the question number before beginning the response.

EXAMINERS

Michael Zaouk (Convenor)	St Ursula's College, Kingsgrove
Stephen D'Arcy	Education Consultant
Dr Alan McCarthy	The University of Notre Dame Australia
Penny Pachos	MLC School, Burwood
Julie Sligar	St Gregory's College, Campbelltown

Centre Number

Student Number

CSSA TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS ADVANCED – MULTIPLE CHOICE ANSWER SHEET

Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you have changed your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ←

A ⊗ B ⊗ C ○ D ○

ATTEMPT ALL QUESTIONS

Question **1** A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

6 A ○ B ○ C ○ D ○

7 A ○ B ○ C ○ D ○

8 A ○ B ○ C ○ D ○

9 A ○ B ○ C ○ D ○

10 A ○ B ○ C ○ D ○

CSSA

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE FROM THE EXAM ROOM

Mathematics Advanced

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

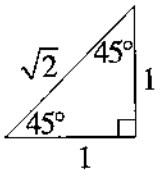
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

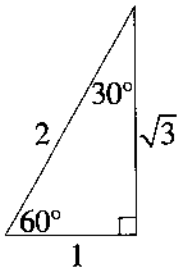
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

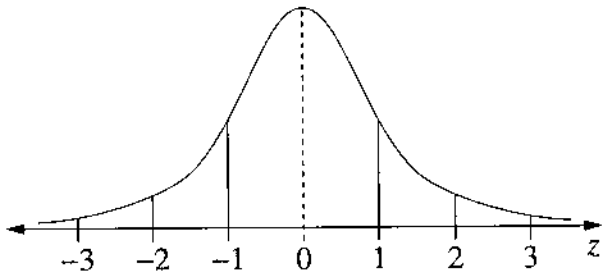
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$