CSSA

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MARKING GUIDELINES

Mathematics Advanced

The Marking Guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Section I
10 marks

Questions 1-10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	A	MA12-1	2-3
2	C	MA12-1	2-3
3	D	MA12-1, MA12-10	3-4
4	B	MA12-8	3-4
5	B	MA12-7	4-5
6	B	MA12-1, MA12-5	4-5
7	C	MA12-1	4-5
8	B	MA12-4	4-5
9	A	MA12-3, MA12-6	5-6
10	A	MA12-3	5-6

Section II
90 marks

Question 11 (2 marks)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 2-3

Criteria	Marks
• Provides two correct solutions	2
• Provides one correct solution, or equivalent merit	1

Sample Answer:

Case 1:

$4x + 1 = 9$

$4x = 8 \therefore x = 2$

Case 2:

$4x + 1 = -9$

$4x = -10 \therefore x = \frac{-5}{2}$

$\therefore x = \frac{-5}{2}, 2$

Question 12 (3 marks)
Outcomes Assessed: MA12-6
Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	3
• Attempts to use the chain rule to differentiate $y = h(x)$	2
• States correct expression for the composition $h(x) = f(g(x))$	1

Sample Answer:

$h(x) = \sqrt{4 - x^2}$

$= (4 - x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(4 - x^2)^{\frac{-1}{2}} \times (-2x)$

$= \frac{-x}{\sqrt{4 - x^2}}$

Question 13

(a) (1 mark)

Outcomes Assessed: MA12-1

Targeted Performance Bands: 2-3

Criteria	Mark
• Provides correct answer	1

Sample Answer:

$$P(A) = \frac{2}{7}$$

(b) (2 marks)

Outcomes Assessed: MA12-1

Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	2
• Determines either $P(\bar{A}A)$ or $P(A\bar{A})$, or calculates probability with replacement	1

Sample Answer:

$$P(\text{first card is A and second card is not A}) = \frac{2}{7} \times \frac{5}{6}$$
$$= \frac{5}{21}$$

$$P(\text{first card is not A and second card is A}) = \frac{5}{7} \times \frac{2}{6}$$
$$= \frac{5}{21}$$

$$P(\text{only one card chosen is A}) = \frac{5}{21} + \frac{5}{21}$$
$$= \frac{10}{21}$$

Question 14 (3 marks)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	3
• Finds the arc length of the shaded region, or equivalent merit	2
• Finds the radius of the circle, or equivalent merit	1

Sample Answer:

Let radius $r = OX = OZ$.

$\frac{1}{2} \times r^2 \times \frac{2\pi}{3} = 12\pi$

hence $r = 6$ cm

$$\text{Arc length of shaded region} = \frac{4\pi}{3} \times 6$$
$$= 8\pi \text{ cm}$$
$$\text{Perimeter of shaded region} = OX + OZ + 8\pi$$
$$= 6 + 6 + 8\pi$$
$$= 12 + 8\pi \text{ cm}$$

Question 15 (3 marks)
Outcomes Assessed: MA12-1, MA12-10
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	3
• Solves quadratic equation and finds two non-integer solutions for n	2
• Forms quadratic equation to find break-even points, or equivalent merit	1

Sample Answer:

Solve $3n^2 + 100 = 80n$

$3n^2 - 80n + 100 = 0$

$$n = \frac{80 \pm \sqrt{(-80)^2 - 4 \times 3 \times 100}}{2 \times 3}$$

$n = 1.315, 25.352$

Katrina will make a profit when $I > C$ and this occurs for $2 \leq n \leq 25$ where n represents the number of computers refurbished and sold.

Hence, the minimum number is 2 and the maximum number is 25 in order to make a profit.

Question 16

(a) (3 marks)

Outcomes Assessed: MA12-8, MA12-9

Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	3
• Determines the equation of the least-squares regression line	2
• Provides an equation with either a correct gradient or y-intercept	1

Sample Answer:

$P = -2285A + 28460$ (using STAT mode on the calculator)

When $A = 4$:

$$P = -2285 \times 4 + 28460$$
$$= 19\,320$$

The cost of a car that is 4 years old is \$19 320

(b) (1 mark)

Outcomes Assessed: MA12-8

Targeted Performance Bands: 2-3

Criteria	Mark
• Provides correct explanation	1

Sample Answer:

Estimation of the value of a 50-year-old car using the least-squares regression line involves extrapolation beyond the range of data given, hence it cannot be used.

Question 17 (3 marks)
Outcomes Assessed: MA12-8
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	3
• Forms correct equation for $E(X) = 2.8$ involving p and q and attempts to solve simultaneously with equation for sum of $P(X = x) = 1$	2
• Forms correct equation for sum of $P(X = x) = 1$	1

Sample Answer:

Sum of probabilities equals 1:

$0.05 + p + p + q + 0.01 = 1$
$2p + q = 0.94$ **(1)**

$E(X) = 2.08$:

$(0 \times 0.05) + (1 \times p) + (2 \times p) + (3 \times q) + (4 \times 0.01) = 2.08$
$3p + 3q = 2.04$
$p + q = 0.68$ **(2)**

Solve **(1)** and **(2)** simultaneously:

From **(1)**: $q = 0.94 - 2p$. Substitute for q in **(2)**
$p + 0.94 - 2p = 0.68$
$-p = -0.26$
$\therefore p = 0.26$ and $q = 0.42$

Question 18 (3 marks)
Outcomes Assessed: MA12-8
Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	3
• Uses empirical rule to determine the percentage corresponding to the range of sizes, or equivalent merit	2
• Finds z-scores corresponding to size 5 and size 8, or equivalent merit	1

Sample Answer:

z-score corresponding to size 5:

$$z = \frac{5-8}{1}$$
$$= -3$$

z-score corresponding to size 10:

$$z = \frac{10-8}{1}$$
$$= 2$$

By the empirical rule, the area under the standard normal curve from $z = -3$ to $z = 2$ is:

$$\frac{99.7\%}{2} + \frac{95\%}{2} = 97.35\%$$

$$4000 \times 97.35\% = 3894$$

3894 women in the town can be expected to buy shoes that fit them.

Question 19

(a) (2 marks)

Outcomes Assessed: MA12-8

Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	2
• Finds $1.5 \times$ IQR, equivalent merit	1

Sample Answer:

$Q_1 = 511\,000, Q_3 = 566\,000$

$IQR = 566\,000 - 511\,000$

$= 55\,000$

$1.5 \times IQR = 82\,500$

Upper bound for outlier is $566\,000 + 82\,500 = 648\,500$

$980\,000 > 648\,500$ hence the most expensive house sold is an outlier.

(b) (2 marks)

Outcomes Assessed: MA12-8

Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct explanation justified with relevant calculations	2
• Provides correct explanation without justification, or equivalent merit	1

Sample Answer:

With the outlier present, median is \$528 000 and mean is \$583 750

With the outlier removed, median is \$528 000 and mean is \$527 143 (to the nearest dollar).

The presence of the outlier significantly increases the mean without any change in the median.

Question 20

(a) (1 mark)

Outcomes Assessed: MA12-1

Targeted Performance Bands: 2-3

Criteria	Mark
• Provides correct solution	1

Sample Answer:

Show that $-f(x) = f(-x)$

$-f(x) = -2x + \sin x$

$$\begin{aligned} f(-x) &= 2(-x) - \sin(-x) \\ &= -2x - -\sin(x) \\ &= -2x + \sin x \\ &= -f(x) \end{aligned}$$

$\therefore f(x)$ is an odd function.

(b) (2 marks)

Outcomes Assessed: MA12-7

Targeted Performance Bands: 3-4

Criteria	Marks
• Provides correct solution	2
• Determines the definite integral without considering the signed area, or equivalent merit	1

Sample Answer:

Using the symmetry property of $f(x)$ from part (a):

$$\begin{aligned} \text{Area} &= 2\int_0^{\pi} 2x - \sin x \, dx \\ &= 2[x^2 + \cos x]_0^{\pi} \\ &= 2\big((\pi^2 + \cos\pi) - (0^2 + \cos 0)\big) \\ &= 2(\pi^2 - 2) \\ &= 2\pi^2 - 4 \end{aligned}$$

Question 21 (2 marks)
Outcomes Assessed: MA12-5
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Provides one correct solution, or equivalent merit	1

Sample Answer:

Solve for x when $y = 0$:

$$3\cos\left(\frac{5x-1}{2}\right) = 0$$
$$\cos\left(\frac{5x-1}{2}\right) = 0$$
$$\left(\frac{5x-1}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$5x - 1 = \pi, 3\pi, 5\pi, \ldots$$
$$\therefore x = \frac{\pi+1}{5}, \frac{3\pi+1}{5}$$

Question 22 (5 marks)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 5

Criteria	Marks
• Provides correct solution	5
• Determines the length of the other sides of triangle ABC using sine or cosine rule	4
• Finds all angles in triangle ABC and distance AB or equivalent merit	3
• Finds both distance AB and angle DAB, or equivalent merit	2
• Finds either distance AB or angle DAB, or equivalent merit	1

Sample Answer:

By Pythagoras' theorem: $AB = \sqrt{85^2 + 33^2} = 91.181\ldots$

Angle $DAB = \tan^{-1}\left(\frac{33}{85}\right) = 21°13'$

Angle $CAB = 90° - 26° - 21°13' = 42°47'$

Angle $ACB = (360° - 321°) + 26° = 65°$ (angles at a point and alternate angles)

Angle $ABC = 180° - 65° - 42°47' = 72°13'$ (angle sum of triangle ABC)

By the sine rule:

$$AC = \frac{\sin 72°13' \times 91.181}{\sin 65°} = 95.800, \qquad BC = \frac{\sin 42°47' \times 91.181}{\sin 65°} = 68.335$$

(Alternatively, BC may be determined using the cosine rule):

$$BC = \sqrt{91.181^2 + 95.800^2 - 2 \times 91.181 \times 68.335 \times \cos 42°47'}$$

$$= 68.335$$

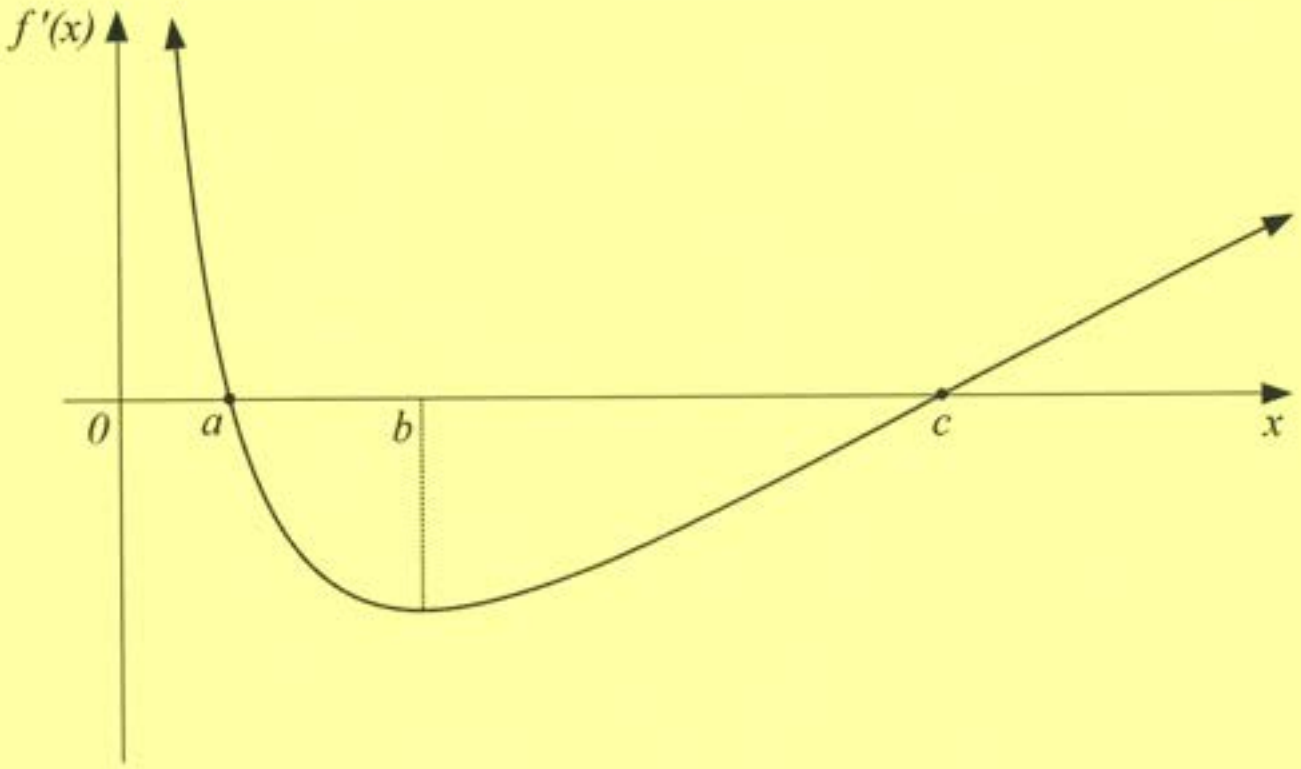
Perimeter of triangle ABC: $91.181 + 95.800 + 68.335 = 255.316$

The total distance travelled is 255 km, correct to the nearest kilometre.

Question 23 (2 marks)
Outcomes Assessed: MA12-6
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Partially correct graph, e.g. incorrect shape but correctly shows points that correspond to the stationary points of $y = f(x)$, or equivalent merit	1

Sample Answer:



Question 24

(a) (2 marks)

Outcomes Assessed: MA12-8

Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	2
• States correct equation and attempts to solve for k, or equivalent merit	1

Sample Answer:

Solve for k:

$$k\int_0^{\pi}\sec^2\left(\frac{x}{3}\right)dx = 1$$

$$3k\left[\tan\left(\frac{x}{3}\right)\right]_0^{\pi} = 1$$

$$3k\left(\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{0}{3}\right)\right) = 1$$

$$3k \times \sqrt{3} = 1$$

$$\therefore k = \frac{1}{3\sqrt{3}}$$

(b) (2 marks)

Outcomes Assessed: MA12-8

Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	2
• States correct equation attempts to solve for t, or equivalent merit	1

Sample Answer:

From part (a), solve for t:

$$3k\left[\tan\left(\frac{x}{3}\right)\right]_0^{t} = 0.5, \text{where } k = \frac{1}{3\sqrt{3}}$$

$$3 \times \frac{1}{3\sqrt{3}}\left[\tan\left(\frac{x}{3}\right)\right]_0^{t} = 0.5$$

$$\frac{1}{\sqrt{3}}\tan\left(\frac{t}{3}\right) - \frac{1}{\sqrt{3}}\tan\left(\frac{0}{3}\right) = 0.5$$

$$\tan\left(\frac{t}{3}\right) = 0.5 \times \sqrt{3}$$

$$\therefore t = 3\tan^{-1}(0.5 \times \sqrt{3}) = 2.14 \text{ (correct to two decimal places)}$$

Question 25 (3 marks)
Outcomes Assessed: MA12-6
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	3
• Finds the location of the stationary point, or equivalent merit	2
• Finds the first derivative, or equivalent merit	1

Sample Answer:

By the quotient rule:

$$f'(x) = \frac{4\sqrt{x-2} - \frac{4x}{2\sqrt{x-2}}}{x-2}$$

$$f'(x) = \frac{4\sqrt{x-2} - \frac{2x}{\sqrt{x-2}}}{x-2}$$

Solve $f'(x) = 0$ to find any stationary points:

$$4\sqrt{x-2} - \frac{2x}{\sqrt{x-2}} = 0$$

$$4(x-2) - 2x = 0$$

$$4x - 8 - 2x = 0$$

$2x = 8$ hence $x = 4$

$$f(x) = \frac{4 \times 4}{\sqrt{4-2}} = 8\sqrt{2}$$

A stationary point exists at $(4, 8\sqrt{2})$

Using a table of gradients to determine the nature of the stationary point:

x	3	4	5
$f'(x)$	-2	0	0.385
Sign	$-$	0	$+$

Hence the stationary point is a local minimum.

Question 26 (3 marks)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct graph	3
• Shows two or three correct transformations, or equivalent merit	2
• Shows one correct transformation, or equivalent merit	1

Sample Answer:

The graph of $y = f(x)$ is a concave up parabola with vertex at $(1, -1)$.

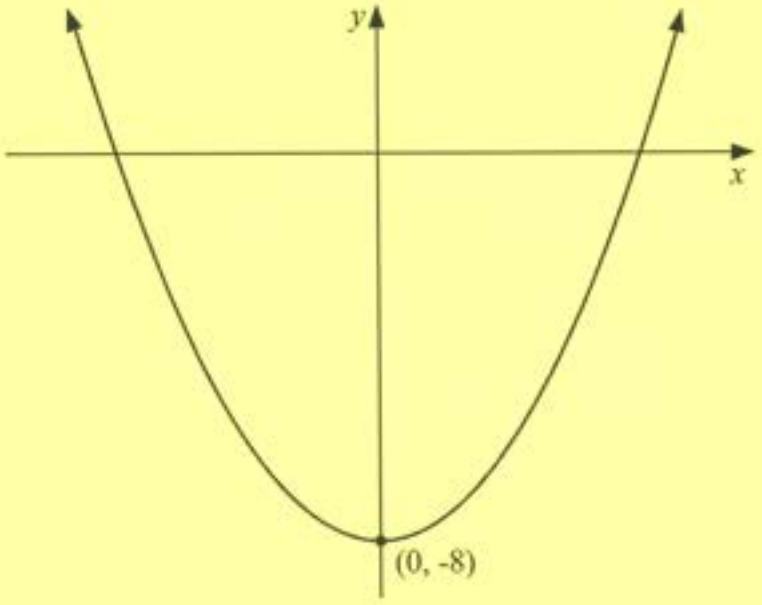
Horizontal transformations:
1. Reflection in the y-axis
2. Translation to the right by 1 unit

Vertical transformations:
1. Dilation by factor 2
2. Translation down by 6 units

x-coordinate of transformed vertex:
$1 \times (-1) + 1 = 0$

y-coordinate of transformed vertex:
$(-1) \times 2 - 6 = -8$

Transformed graph is a concave up parabola, with vertex at $(0, -8)$.



Question 27 (3 marks)
Outcomes Assessed: MA12-7
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	3
• Finds the value of the constant of integration, or equivalent merit	2
• Correctly integrates expression representing rate of fuel consumption, or equivalent merit	1

Sample Answer:

The amount of fuel, $F(t)$ litres, consumed at time t hours is given by

$$F(t) = \int R dt$$

$$F(t) = \int 5 + 4e^{-0.5t}\, dt$$

$$F(t) = 5t - 8e^{-0.5t} + c$$

At time $t = 1$, vehicle has consumed 25 litres of fuel since the start of the journey.
Finding the constant of integration, c:

$$25 = 5(1) - 8e^{-0.5(1)} + c$$

$$20 = -8e^{-0.5} + c$$

$$c = 20 + 8e^{-0.5}$$

At time $t = 4$, the amount of fuel the vehicle consumed is given by

$$F = 5(4) - 8e^{-0.5(4)} + 20 + 8e^{-0.5}$$

$$F = 43.7695 \ldots$$

After 4 hours of travel, the vehicle has consumed 44 litres of fuel.

Question 28

(a) (3 marks)

Outcomes Assessed: MA12-2

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides the correct solution	3
• Finds the interest earned at the end of the fifth month, or equivalent merit	2
• Finds the balance at the end of the fourth month, or equivalent merit	1

Sample Answer:

Balance at the end of the fourth month:

$496994.00 + 1490.98 - 3800 = 494684.98$

∴ Balance at the end of the fourth month is \$494 684.98

Interest earned at the end of the fifth month:

$494684.98 \times 0.003 = 1484.05$

∴ Interest earned at the end of the fifth month is \$1484.05

Amount Maroun withdrew at the end of the fifth month:

Balance $= P + I - M$

$M = P + I - \text{Balance}$

$M = 494684.98 + 1484.05 - 491969.03$

$= 4200$

∴ Maroun withdrew \$4200 at the end of the fifth month

(b) (2 marks)

Outcomes Assessed: MA12-2

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides the correct solution	2
• Determines the balance at the end of the sixth month, or equivalent merit	1

Sample Answer:

$A_5 = 491969.03$

$A_6 = 491969.03 \times 1.003 - 7500$

$= 485944.94$

$A_7 = 485944.94 \times 1.003 - 7500$

$= 479902.77$

∴ Balance at the end of the seventh month is \$479 902.77

Question 28
(c) (2 marks)
Outcomes Assessed: MA12-2
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Finds the effective annual rate of interest for one of the funds, or equivalent merit	1

Sample Answer:

Effective annual rate of interest for the current retirement fund:

$$(1 + 0.003)^{12} - 1 = 0.036599 \ldots$$
$$\approx 3.66\% \text{ p.a.}$$

Effective annual rate of interest for the alternative retirement fund:

$$\left(1 + \frac{0.032}{365}\right)^{365} - 1 = 0.032516 \ldots$$
$$\approx 3.25\% \text{ p.a.}$$

The effective annual rate of interest for the alternative retirement fund is less than Maroun's current retirement fund, hence Maroun should stay with his current retirement fund.

Alternatively, compare the future value (FV) of one dollar invested in both funds.

Current fund: $\text{FV} = (1 + 0.3\%)^{12} = \$1.03659998$

Alternative fund: $\text{FV} = \left(1 + \frac{3.2\%}{365}\right)^{365} = \$1.032516057$

The FV of the alternative fund is less than the FV of the current fund, hence Maroun should stay with his current retirement fund.

Question 29

(a) (2 marks)

Outcomes Assessed: MA12-3, MA12-6

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Correctly differentiates to find an equation for speed, or equivalent merit	1

Sample Answer:

$$S'(t) = \frac{dS}{dt} = -0.182 \times \ln 3 \times 55000 \times 3^{-0.182t}$$
$$= -10010 \ln 3 \times 3^{-0.182t}$$

Speed of the rocket at time $t = 2$:

$$S'(2) = |-10010 \ln 3 \times 3^{-0.182 \times 2}|$$
$$= -7372.3576 \ldots$$
$$\approx 7372$$

The speed of the rocket 2 hours after it begins its return journey is 7372 km/h

(b) (2 marks)

Outcomes Assessed: MA12-1

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Attempts to solve for t, or equivalent merit	1

Sample Answer:

Let $S = 1000$ and solve for t the equation: $1000 = 55000 \times 3^{-0.182t}$

$$\frac{1}{55} = 3^{-0.182t}$$

$$\ln\left(\frac{1}{55}\right) = -0.182t \times \ln 3$$

$$t = \ln\left(\frac{1}{55}\right) \div (-0.182 \times \ln 3)$$
$$= 20.0419 \ldots$$
$$\approx 20.04$$

∴ Rocket deploys its parachute 20.04 hours after it begins its return journey

Question 30 (3 marks)
Outcomes Assessed: MA12-7
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	3
• Finds the area using integration and the trapezoidal rule, or equivalent merit	2
• Finds the area using either the trapezoidal rule or integration, or equivalent merit	1

Sample Answer:

Exact area under the curve:

$$\text{Area} = \int_0^3 x^3 - 3x^2 + 3x + 4\,dx$$

$$= \left[\frac{x^4}{4} - x^3 + \frac{3x^2}{2} + 4x\right]_0^3$$

$$= \frac{75}{4} \text{ or } 18.75$$

Estimate of the area under the curve using two applications of the trapezoidal rule:

$$\text{Area} \approx \frac{1.5}{2}[4 + 2 \times 5.125 + 13]$$
$$\approx 20.4375$$

$$20.4375 - 18.75 = 1.6875$$

Hence the trapezoidal rule overestimates the exact area under the curve by 1.6875 square units.

Question 31

(a) (2 marks)

Outcomes Assessed: MA12-4

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Attempts to model height of bounce as a geometric progression, or equivalent merit	1

Sample Answer:

Initial height of the ball is H metres. The height after the nth bounce is the nth term of the geometric progression with first term $a = H$ and common ratio $r = \frac{q}{100}, 0 < q < 100$.

Height after the second bounce:

$$h_n = T_n = ar^{n-1}$$
$$h_2 = H \times \left(\frac{q}{100}\right)^2$$
$$= \frac{q^2 H}{(100)^2}$$

(b) (3 marks)

Outcomes Assessed: MA12-4

Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	3
• Attempts to simplify the expression involving the limiting sum, or equivalent merit	2
• Correctly models the total distance involving the limiting sum, or equivalent merit	1

Sample Answer:

Total distance travelled may be modelled as the limiting sum of a geometric series, since $0 < r < 1$.

$$S_\infty = H + 2(h_1 + h_2 + h_3 + \cdots)$$
$$= H + 2\left(\frac{\frac{qH}{100}}{1 - \frac{q}{100}}\right)$$
$$= H + \frac{2qH}{100 - q} \rightarrow = \frac{H(100 - q)}{100 - q} + \frac{2qH}{100 - q}$$
$$= \frac{100H - qH + 2qH}{100 - q} \rightarrow = \frac{100H + qH}{100 - q}$$
$$= \frac{H(100 + q)}{100 - q}$$

Question 32

(a) (1 mark)

Outcomes Assessed: MA12-3, MA12-6

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the equation y in terms of x, given the area	1

Sample Answer:

$6 \times (2y + 8x) = 120$

$2y + 8x = 20$

$y + 4x = 10$

$\therefore y = 10 - 4x$

(b) (2 marks)

Outcomes Assessed: MA12-3, MA12-6

Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Attempts to use Pythagoras' theorem to find h, or equivalent merit	1

Sample Answer:

$$h^2 = (4x)^2 - \left(\frac{5x}{2}\right)^2$$

$$= 16x^2 - \frac{25x^2}{4}$$

$$= \frac{64x^2}{4} - \frac{25x^2}{4}$$

$$= \frac{39x^2}{4}$$

$$h = \sqrt{\frac{39x^2}{4}}$$

$$= \frac{\sqrt{39}x}{2}$$

Question 32

(c) (3 marks)

Outcomes Assessed: MA12-3, MA12-6

Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	3
• Finds stationary point, or equivalent merit	2
• Finds correct expression for surface area of tent, or equivalent merit	1

Sample Answer:

Maximise the volume by maximising the area of the cross-section, since the depth is constant.
Let A be the area of the cross-section of the tent.

$$A = 5xy + \frac{1}{2} \times 5x \times h$$
$$= 5xy + \frac{5xh}{2}$$

From parts (a) and (b):

$$A = 5x(10 - 4x) + \frac{5\sqrt{39}x^2}{4}$$
$$= 50x - 20x^2 + \frac{5\sqrt{39}x^2}{4}$$
$$= 50x - \left(20 - \frac{5\sqrt{39}}{4}\right)x^2$$

Determine stationary point and its nature:

$$\frac{dA}{dx} = 50 - 2\left(20 - \frac{5\sqrt{39}}{4}\right)x$$

$$50 - 2\left(20 - \frac{5\sqrt{39}}{4}\right)x = 0$$

$$\left(20 - \frac{5\sqrt{39}}{4}\right)x = 25$$

$$x = 25 \div \left(20 - \frac{5\sqrt{39}}{4}\right) = 2.05$$

$$\frac{d^2A}{dx^2} = -2\left(20 - \frac{5\sqrt{39}}{4}\right) = -24.387 \ldots < 0$$

Hence maximum volume at $x = 2.05$

Question 32
(d) (2 marks)
Outcomes Assessed: MA12-3, MA12-6
Targeted Performance Bands: 4-5

Criteria	Marks
• Provides correct solution	2
• Attempts to calculate the volume using values of x, y, h or using the area of the cross-section from part (c), or equivalent merit	1

Sample Answer:

Maximum volume of tent = maximum area of cross-section × depth

From part (c):

$$V = 6 \times \left(50x - \left(20 - \frac{5\sqrt{39}}{4}\right)x^2\right), \quad x = 2.05$$

$$V = 6 \times \left(50 \times 2.05 - \left(20 - \frac{5\sqrt{39}}{4}\right) \times 2.05^2\right)$$

$$= 307.5345 \ldots$$

$$= 307.5 \text{ m}^3$$

Question 33 (4 marks)
Outcomes Assessed: MA12-3, MA12-7
Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	4
• Correctly sets up a sum of two integrals, including correct bounds for each integral, and attempts to find the area between the two curves, or equivalent merit	3
• Finds the x-coordinates of A and B, or equivalent merit	2
• Attempts to find the x-coordinates of A and B by equating the functions, or equivalent merit	1

Sample Answer:

Find the x-coordinates of A and B by equating $y = x^3 - 2x$ with $y = x^2$.

$x^3 - 2x = x^2$
$x^3 - x^2 - 2x = 0$
$x(x^2 - x - 2) = 0$
$x(x + 1)(x - 2) = 0$
hence the x-coordinate of A is -1 and the x-coordinate of B is 2.

Area between the two curves is given by

$$\text{Area} = \left|\int_{-1}^{0} x^3 - x^2 - 2x \ dx\right| + \left|\int_{0}^{2} x^3 - x^2 - 2x \ dx\right|$$
$$= \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0}\right| + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{0}^{2}\right|$$
$$= \left|\frac{1}{4} + \frac{1}{3} - 1\right| + \left|4 - \frac{8}{3} - 4\right|$$
$$= \frac{5}{12} + \frac{8}{3}$$
$$= \frac{37}{12}$$

Question 34 (3 marks)
Outcomes Assessed: MA12-1, MA12-10
Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	3
• Substantial progress towards correct solution	2
• Uses Pythagorean identities, reciprocal ratios or complementary trigonometric ratios of $\cos(90° - \theta)$ and $\tan(90° - \theta)$, or equivalent merit	1

Sample Answer:

$$\begin{aligned}
\text{LHS} &= \frac{\cos\theta}{\cos(90^0 - \theta)}(1 + \tan^2\theta) + \frac{\sin\theta}{\cos\theta}[1 + \tan^2(90^0 - \theta)] \\
&= \frac{\cos\theta}{\sin\theta} \times \sec^2\theta + \frac{\sin\theta}{\cos\theta}(1 + \cot^2\theta) \\
&= \frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \text{cosec}^2\theta \\
&= \frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin^2\theta} \\
&= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\sin\theta\cos\theta} \\
&= 2\text{cosec}\,\theta\sec\theta \\
&= \text{RHS}
\end{aligned}$$

Question 35 (3 marks)
Outcomes Assessed: MA12-7
Targeted Performance Bands: 5-6

Criteria	Marks
• Provides correct solution	3
• Makes substantial progress toward correct solution, such as recognising that the integral is in the form $\int \frac{f'(x)}{f(x)}$ and hence will result in a log function by using the fact that $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1-\ln x}{x^2}$, or equivalent merit	2
• Divides numerator and denominator by x^2, or equivalent merit	1

Sample Answer:

Divide the numerator and denominator of the integrand by x^2 such that:

$$\int \frac{1-\ln x}{x\ln x}\,dx = \int \frac{\frac{1-\ln x}{x^2}}{\frac{x\ln x}{x^2}}\,dx$$

$$= \int \frac{\frac{1-\ln x}{x^2}}{\frac{\ln x}{x}}\,dx$$

Using $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1-\ln x}{x^2}$ and $\int \frac{f'(x)}{f(x)}\,dx = \ln x + c$:

$\int \frac{\frac{1-\ln x}{x^2}}{\frac{\ln x}{x}}\,dx$ is in the form $\int \frac{f'(x)}{f(x)}\,dx$

Hence,

$$\int \frac{1-\ln x}{x\ln x}\,dx = \ln\left|\frac{\ln x}{x}\right| + c$$

$$= \ln|\ln x| - \ln|x| + c$$

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