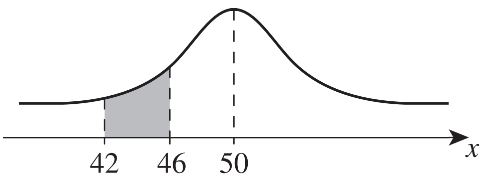
Neap

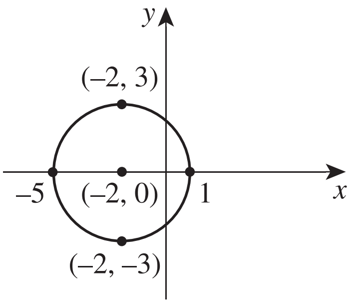
Trial Examination 2021

HSC Year 12 Mathematics Advanced

Solutions and marking guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **D** **D** is correct. A probability density function defined on a closed interval $[a, b]$ needs to satisfy the following conditions. • $f(x) \geq 0$ for $[a, b]$ • $\int_a^b f(x)\,dx = 1$ The graph shown in **D** has part of the graph below the x-axis and thus does not satisfy the condition $f(x) \geq 0$. **A**, **B** and **C** are incorrect. As these options satisfy the conditions above, they could represent probability density functions.	MA–S3 Random Variables MA12–10 Band 2
Question 2 **B** For compounded half-yearly (2 times a year): $r = \frac{2\%}{2}$ $= 1\%$ (per half-year) $n = 4 \times 2$ $= 8$ From the table, the interest factor is 8.2857. future value = present value × interest factor $34\,800$ = present value × 8.2857 present value = \$4200	MA–M1 Modelling Financial Situations MA12–2 Bands 3–4
Question 3 **A** $\mu = 50$ and $\sigma = 4$. $X = 42$ is two standard deviations below the mean. $X = 46$ is one standard deviation below the mean.  $P(42 < \text{amount} < 46) = \frac{95-68}{2}$ $= 13.5\%$	MA–S3 Random Variables MA12–8 Bands 3–4

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 4 **B** $P\left(\text{fatal} \mid \overline{\text{alcohol consumption}}\right)$ $= \dfrac{P\left(\text{fatal} \cap \overline{\text{alcohol consumption}}\right)}{P\left(\overline{\text{alcohol consumption}}\right)}$ $= \dfrac{35+12}{35+75+12+55}$ $= \dfrac{47}{177}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4
Question 5 **D** Completing the square: $x^2+4x+4+y^2=5+4$ $(x+2)^2+y^2=9$ The diagram shows a circle with centre (–2, 0) and radius of 3.  domain $= [-5,\ 1]$; range $= [-3,\ 3]$	MA–F1 Working with Functions MA11–1 Bands 3–4
Question 6 **D** The amplitude is 2; hence, the maximum value of $y=2\sin\left(\dfrac{x}{3}\right)+1$ is 3. When $y = 3$: $3=2\sin\left(\dfrac{x}{3}\right)+1$ $1=\sin\left(\dfrac{x}{3}\right)$ $\dfrac{x}{3}=\dfrac{\pi}{2}$ $x=\dfrac{3\pi}{2}$	MA–T3 Trigonometric Functions and Graphs MA12–5 Band 4

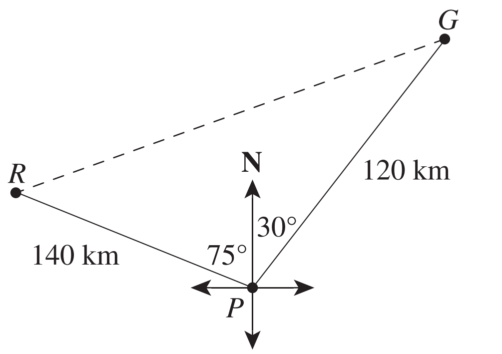
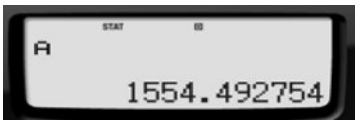
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 **B** **B** is correct. The graph depicts the correct properties. The graph of $y = f(x)$ is: • increasing in the domain $(-\infty, 0)$ • decreasing in the domain $(0, \infty)$ • stationary at $x = 0$ • as $x \to \pm\infty$, $y \to 1$. Hence, the derivative $y = f'(x)$ must have the following properties. • above the x-axis in the domain $(-\infty, 0)$ • below the x-axis in the domain $(0, \infty)$ • x-intercept at $x = 0$ • as $x \to \pm\infty$, $f'(x) \to 0$ **A** is incorrect because an x-intercept does not exist at $x = 0$. **C** is incorrect because the graph is below the x-axis as $x \to -\infty$. **D** is incorrect because as $x \to \pm\infty$, the graph is not approaching the x-axis.	MA–C1 Introduction to Differentiation MA12–5 Band 4
Question 8 **B** $\int_2^6 f(x)\,dx = 3$ As $f(2x)$ is a horizontal dilation of $f(x)$, the graph is compressed horizontally by a factor of $\frac{1}{2}$. This means that the range is unchanged but the domain is halved, thereby halving the area under the graph. $\int_1^3 f(2x)\,dx = \frac{1}{2} \times \int_2^6 f(x)\,dx$ $= \frac{3}{2}$ Horizontally translating the graph three units to the right gives: $\int_4^6 f\big(2(x-3)\big)\,dx = \frac{3}{2}$	MA–C4 Integral Calculus MA–F2 Graphing Techniques MA12–10 Band 5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **C** **C** is correct. Let $h(x)=f\left[g(x)\right]$. $h(-x)=f\left[g(-x)\right]$ $=f\left[-g(x)\right]$ (since $g(x)$ is odd) $=f\left[g(x)\right]$ (since $f(x)$ is even) Therefore, $h(x)=h(-x)\Rightarrow f\left[g(x)\right]$ is an even function. **A** is incorrect. Let $h(x)=f(x)\times g(x)$. $h(-x)=f(-x)\times g(-x)$ $=f(x)\times -g(x)$ Therefore, $h(x)\neq h(-x)\Rightarrow f(x)\times g(x)$ is not an even function. **B** is incorrect. Let $h(x)=f(x)+g(x)$. $h(-x)=f(-x)+g(-x)$ $=f(x)-g(x)$ $-h(x)=-f(x)-g(x)$ Therefore, $h(-x)\neq -h(x)\Rightarrow f(x)+g(x)$ is not an odd function. **D** is incorrect. Let $h(x)=g\left[f(x)\right]$. $h(-x)=g\left[f(-x)\right]$ $=g\left[f(x)\right]$ $-h(x)=-g\left[f(x)\right]$ Therefore, $h(-x)\neq -h(x)\Rightarrow g\left[f(x)\right]$ is not an odd function.	MA–F1 Working with Functions MA11–9 Bands 5–6

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 10 **C** Since $\Sigma p(x)=1$: $\frac{1}{10}+a+b+b+2b=1$ $a+4b=\frac{9}{10}$ $a=\frac{9}{10}-4b$ The maximum value of b occurs when $a=0$. $0=\frac{9}{10}-4b$ $4b=\frac{9}{10}$ $b=\frac{9}{40}$ $0\le b\le\frac{9}{40}$ $E(X)=\Sigma xp(x)$ $=\left(-1\times\frac{1}{10}\right)+(0\times a)+(1\times b)+(a\times b)+(2a\times 2b)$ $=-\frac{1}{10}+b+5ab$ $=-\frac{1}{10}+b+5b\left(\frac{9}{10}-4b\right)$ $=-\frac{1}{10}+b+\frac{45b}{10}-20b^2$ $=-\frac{1}{10}+\frac{11}{2}b-20b^2$ for $0\le b\le\frac{9}{40}$ For the smallest value of $E(X)$, $b=0$: $E(X)=-\frac{1}{10}$ For the largest value of $E(X)$, find the maximum value of the quadratic $E(X)=-\frac{1}{10}+\frac{11}{2}b-20b^2$: $b=\frac{-\left(\frac{11}{2}\right)}{2(-20)}$ $=\frac{11}{80}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 6

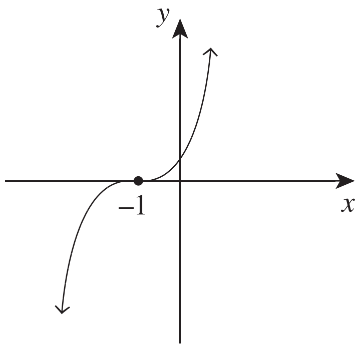
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 10 (continued) When $b=\frac{11}{80}$: $E(X)=-\frac{1}{10}+\frac{11}{2}\left(\frac{11}{80}\right)-20\left(\frac{11}{80}\right)^2$ $=\frac{89}{320}$ Therefore, $-\frac{1}{10}\le E(X)\le\frac{89}{320}$.	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 6

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a)  $\angle GPR = 75° + 30°$ $= 105°$	MA–T1 Trigonometry and Measure of Angles MA11–9 Bands 2–3 • Gives the correct solution 1
(b) After 2 hours, Greg has travelled a distance of 120 km and Ringo has travelled a distance of 140 km. Let d be the distance apart. $d^2 = 140^2 + 120^2 - 2(140)(120)\cos 105°$ $= 42\,696.31992\ldots$ $d = 206.63$ ≈ 207 km Therefore, they are 207 km apart.	MA–T1 Trigonometry and Measure of Angles MA11–1 Bands 3–4 • Gives the correct solution .... 2 • Attempts to use the cosine rule to find the distance 1
Question 12	
The least-squares regression line is of the form $y = Bx + A$.   $y = 34.64x + 1554.49$ When $x = 5$: $y = 34.64 \times 5 + 1554.49$ $= 1727.69$ Therefore, Beth's chess rating is predicted to be 1728.	MA–S2 Descriptive Statistics and Bivariate Data Analysis MA12–9 Bands 3–4 • Gives the correct solution .... 3 • Correctly provides the equation of the least-squares regression line .... 2 • Correctly provides constant A OR constant B 1

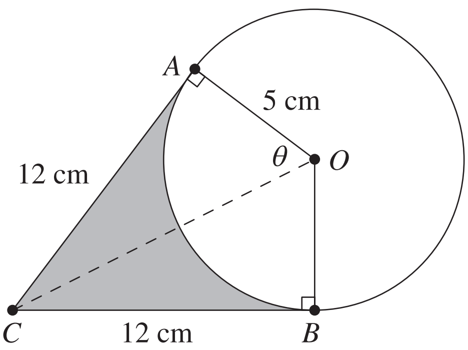
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	
(a) $N = m \times P + C$ When $P = 50$, $N = 12\,500$: $12\,500 = 50m + C$ (1) When $P = 35$, $N = 14\,000$: $14\,000 = 35m + C$ (2) (2) – (1): $1500 = -15m$ $m = -100$ Substitute $m = -100$ into (1): $12\,500 = 50(-100) + c$ $= -5000 + c$ $c = 17500$ $\therefore N = -100P + 17\,500$	MA–F1 Working with Functions MA11–2 Bands 2–4 • Gives the correct solution . . . . 3 • Correctly solves the equations simultaneously to obtain either m OR c. . . . . 2 • Correctly develops and attempts to solve the simultaneous equations 1
(b) $R = (-100P + 17500) \times P$ $= -100P^2 + 17\,500P$ Maximum revenue generated is calculated by finding the maximum value of $R = -100P^2 + 17\,500P$. This occurs at the turning point: $x = -\dfrac{b}{2a}$ $P = \dfrac{-17\,500}{2(-100)}$ $= 87.5$ When $P = 87.5$: $R = -100(87.5)^2 + 17\,500(87.5)$ $= \$765\,625$	MA–F1 Working with Functions MA11–2 Bands 2–4 • Gives the correct solution . . . . 2 • Correctly finds quadratic equation that represents the revenue. 1
(c) When $N = 15\,000$, the price of the ticket would be: $15\,000 = -100P + 17\,500$ $P = 25$ Hence, the revenue generated would be: $R = -100(25)^2 + 17\,500(25)$ $= \$375\,000$ $\text{revenue loss} = 765\,625 - 375\,000$ $= \$390\,625$	MA–F1 Working with Functions MA11–9 Bands 2–4 • Gives the correct solution . . . . 2 • Correctly finds the price when $N = 15\,000$ 1

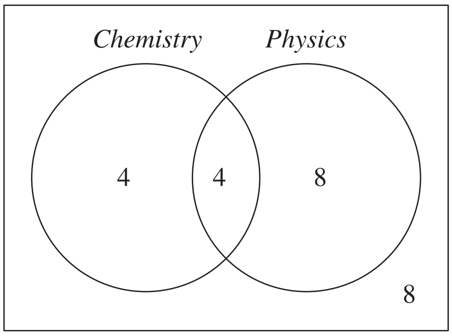
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
$y=\dfrac{1}{4x+1}$ $=(4x+1)^{-1}$ $y'=-4(4x+1)^{-2}$ $=\dfrac{-4}{(4x+1)^2}$ When $x=0$: $y'=\dfrac{-4}{(4\times 0+1)^2}$ $=-4$ For the equation of the tangent: $y-y_1=m(x-x_1)$ $y-1=-4(x-0)$ $y=-4x+1$	MA–C1 Introduction to Differentiation MA11–5 Bands 3–4 • Gives the correct solution . . . . 3 • Finds the gradient of the tangent at $x=0$ . . . . 2 • Finds the derivative 1
Question 15	
(a) $y=\dfrac{1}{2}\ln\left(x^2\right)$ $y'=\dfrac{1}{2}\times\dfrac{2x}{x^2}$ $=\dfrac{x}{x^2}$ $=\dfrac{1}{x}$	MA–C2 Differential Calculus MA12–6 Band 3 • Gives the correct solution . . . . 2 • Correctly differentiates $\ln\left(x^2\right)$ . . 1
(b) $y=\dfrac{e^x}{\sin x}$ $y'=\dfrac{\sin x\times e^x-e^x(\cos x)}{\sin^2 x}$ $=\dfrac{e^x(\sin x-\cos x)}{\sin^2 x}$	MA–C2 Differential Calculus MA12–6 Bands 3–4 • Gives the correct solution . . . . 2 • Attempts to use the quotient rule 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 16	
$\int_0^{\frac{\pi}{4}} x + \sin x \, dx = \left[\frac{x^2}{2} - \cos x\right]_0^{\frac{\pi}{4}}$ $= \left[\frac{\left(\frac{\pi}{4}\right)^2}{2} - \cos\frac{\pi}{4}\right] - \left[\frac{0^2}{2} - \cos 0\right]$ $= \left(\frac{\pi^2}{32} - \frac{1}{\sqrt{2}}\right) - (0-1)$ $= \frac{\pi^2}{32} - \frac{1}{\sqrt{2}} + 1$	MA–C4 Integral Calculus MA12–7 Bands 3–4 • Gives the correct solution . . . . 2 • Correctly finds the anti-derivative 1
Question 17	
	MA–C3 Applications of Differentiation MA12–3 Bands 3–4 • Sketches a correct graph showing all THREE of: – When $x < -1$, the graph is increasing and concave down. – When $x > -1$, the graph is increasing and concave up. – When $x = -1$, a horizontal point of inflection exists . . . . . .2 • Sketches a correct graph showing at least ONE of: – When $x < -1$, the graph is increasing and concave down. – When $x > -1$, the graph is increasing and concave up. – When $x = -1$, a horizontal point of inflection exists 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 18	
For $x > 21$: $\log_{10}(x-21) = 2 - \log_{10} x$ $\log_{10}(x-21) + \log_{10} x = 2$ $\log_{10}[x(x-21)] = 2$ $x(x-21) = 10^2$ $x^2 - 21x - 100 = 0$ $(x-25)(x+4) = 0$ $x = -4$ or 25 Since $x > 21$, $x = 25$ is the only solution.	MA–E1 Logarithms and Exponentials MA11–6 Band 4 • Gives the correct solution . . . . 3 • Correctly applies the log laws to obtain the quadratic equation $x^2 - 21x - 100$ . . . . 2 • Correctly applies the log laws 1
Question 19	
$S_{20} = 4S_{10}$ $\frac{20}{2}[2(5)+19d] = 4 \times \frac{10}{2}[2(5)+9d]$ $10(10+19d) = 20(10+9d)$ $10+19d = 2(10+9d)$ $10+19d = 20+18d$ $d = 10$	MA–M1 Modelling Financial Situations MA12–4 Bands 3–4 • Gives the correct solution . . . . 2 • Correctly substitutes known values to obtain the equation $\frac{20}{2}[2(5)+19d]$ $= 4 \times \frac{10}{2}[2(5)+9d]$. 1
Question 20	
(a) $\Sigma p(x) = 1$: $a + \frac{a}{2} + \frac{a}{4} + \frac{a}{8} + \frac{a}{16} + 0 = 1$ $\frac{31a}{16} = 1$ $a = \frac{16}{31}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 3 • Gives the correct solution . . . . 2 • Correctly finds the probabilities for at least ONE of the random variables 1
(b) $E(X) = \Sigma x p(x)$ $= \left(1 \times \frac{16}{31}\right) + \left(2 \times \frac{8}{31}\right) + \left(3 \times \frac{4}{31}\right) + \left(4 \times \frac{2}{31}\right)$ $+ \left(5 \times \frac{1}{31}\right)$ ≈ 1.84 (to 2 decimal places) Shirley's claim is correct. As the expected value is 1.84, over a long period of time Shirley would need around two attempts to successfully start her car.	MA–S1 Probability and Discrete Probability Distributions MA11–9 Bands 3–4 • Correctly calculates the expected value AND links this to Shirley's claim . . . . 2 • Correctly calculates the expected value 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 21	
(a) <table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>$\sqrt{1-x^2}$</td><td>1</td><td>0.968</td><td>**0.866**</td><td>0.661</td><td>0</td></tr></table>	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 3 • Correctly completes the table 1
(b) $A \approx \frac{b-a}{2n}\left[f(a)+f(b)+2\left(f(x_1)+f(x_2)+\ldots+f(x_{n-1})\right)\right]$ $=\frac{1-0}{2(4)}\left[1+0+2\left(0.968+0.866+0.661\right)\right]$ $=0.74875$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Band 3 • Gives the correct solution . . . . 2 • Shows correct progress using the trapezoidal rule 1
(c) By increasing the number of sub-intervals used in part (a), a better approximation of the shaded area could be obtained.	MA–C4 Integral Calculus MA12–10 Band 3 • Gives the correct explanation 1
Question 22	
Vertical dilation: $\text{amplitude} = \frac{3-(-1)}{2} - 2$ Therefore, $k = 2$. Vertical translation: The centre line is $y = 1$. This means the graph has undergone a vertical shift of 1. Therefore, $c = 1$. Horizontal dilation: Notice the period is 4π. Hence: $\frac{2\pi}{a} = 4\pi \Rightarrow a = \frac{1}{2}$ $\therefore\ a = \frac{1}{2}$ Horizontal translation: The horizontal translation is to the left by $\frac{\pi}{4}$. Therefore, $b = \frac{\pi}{4}$. The equation of the function is $y = 2\cos\left(\frac{1}{2}\left(x+\frac{\pi}{4}\right)\right)+1$.	MA–T3 Trigonometric Functions and Graphs MA12–5 Band 4 • Gives the correct equation of the function . . . . 3 • Correctly provides at least TWO of the constants . . . . 2 • Correctly provides ONE of the constants 1

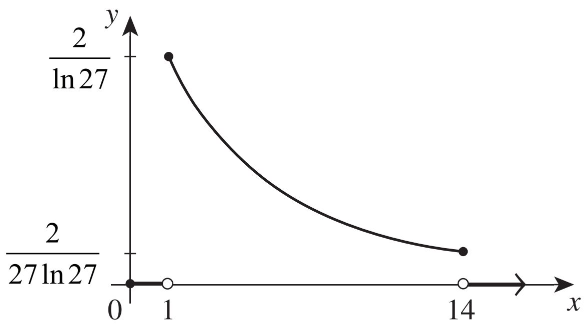
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 23	
(a) English test: $z = \dfrac{x-\mu}{\sigma}$ $= \dfrac{85-60}{22}$ ≈ 1.14 Mathematics test: $z = \dfrac{x-\mu}{\sigma}$ $= \dfrac{75-52}{15}$ ≈ 1.53	MA–S3 Random Variables MA12–8 Bands 3–4 • Gives the correct solution .... 2 • Correctly finds the z-score for ONE subject 1
(b) Relative to the rest of her class, Alison performed slightly better in the Mathematics test. She performed approximately 1.53 standard deviations above the mean in the Mathematics test compared to 1.14 standard deviations above the mean in the English test.	MA–S3 Random Variables MA12–8 Bands 3–4 • Identifies the correct subject and provides a justification 1
Question 24	
(a) Construct the line segment OC and let $\angle AOC = \theta$.  $\tan\theta = \dfrac{12}{5}$ $\theta = \tan^{-1}\left(\dfrac{12}{5}\right)$ $\therefore \angle AOB = 2 \times \tan^{-1}\left(\dfrac{12}{5}\right)$ $= 2.35$ ≈ 2.4 (to 2 significant figures)	MA–T1 Trigonometry and Measure of Angles MA11–3 Bands 3–4 • Gives the correct solution .... 2 • Shows progress towards the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $A_{\text{sector } AOB} = \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 5^2 \times 2.4$ $= 30$ $A_{AOBC} = 2 \times \left(\frac{1}{2} \times 5 \times 12\right)$ $= 60$ $A_{\text{shaded region}} = 60 - 30$ $= 30 \text{ cm}^2$	MA–T1 Trigonometry and Measure of Angles MA11–3 Band 4 • Gives the correct solution .... 2 • Correctly calculates the area of sector *AOB* OR the area of *AOBC* 1
Question 25	
(a) $\lvert \text{CHE} \cup \text{PHY} \rvert = \lvert \text{CHE} \rvert + \lvert \text{PHY} \rvert - \lvert \text{CHE} \cap \text{PHY} \rvert$ $16 = 8 + 12 - \lvert \text{CHE} \cap \text{PHY} \rvert$ $\lvert \text{CHE} \cap \text{PHY} \rvert = 20 - 16$ $= 4$ 	MA–S1 Probability and Discrete Probability Distributions MA11–8 Band 3 • Draws the correct diagram....... 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $P(\text{PHY}) = \frac{12}{24}$ $= \frac{1}{2}$ $P(\text{CHE}) = \frac{8}{24}$ $= \frac{1}{3}$ $P(\text{CHE} \cap \text{PHY}) = \frac{4}{24}$ $= \frac{1}{6}$ $P(\text{PHY}) \times P(\text{CHE}) = \frac{1}{2} \times \frac{1}{3}$ $= \frac{1}{6}$ $= P(\text{CHE} \cap \text{PHY})$ As $P(\text{CHE}) \times P(\text{PHY}) = P(\text{CHE} \cap \text{PHY})$, the two events are independent.	MA–S1 Probability and Discrete Probability Distributions MA11–8 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly calculates either $P(\text{CHE} \cap \text{PHY})$ OR $P(\text{CHE}) \times P(\text{PHY})$ OR equivalent condition for independence 1
(c) $P(\text{CHE}) = \frac{1}{3}$, $P(\text{PHY}) = \frac{2}{5}$ and $P(\text{PHY} \mid \text{CHE}) = \frac{3}{7}$. $P(\text{CHE} \cup \text{PHY}) = P(\text{CHE}) + P(\text{PHY}) - P(\text{CHE} \cap \text{PHY})$ $= \frac{1}{3} + \frac{2}{5} - \left(P(\text{CHE}) \times P(\text{CHE} \mid \text{PHY})\right)$ $= \frac{1}{3} + \frac{2}{5} - \left(\frac{1}{3} \times \frac{3}{7}\right)$ $= \frac{62}{105}$	MA–S1 Probability and Discrete Probability Distributions MA11–8 Bands 5–6 • Gives the correct solution . . . . 2 • Correctly calculates $P(\text{CHE} \cap \text{PHY})$. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 26	
(a) For the displacement function: $x = \int 8\cos\left(2t - \frac{\pi}{2}\right)dt$ $= 8\left[\frac{1}{2}\sin\left(2t - \frac{\pi}{2}\right)\right] + C$ $= 4\sin\left(2t - \frac{\pi}{2}\right) + C$ When $t = 0$, $x = 4$: $4 = 4\sin\left(-\frac{\pi}{2}\right) + C$ $4 = -4 + C$ $C = 8$ $\therefore x = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	MA–C4 Integral Calculus MA12–3 Bands 3–4 • Gives the correct solution . . . . 2 • Correctly finds the anti-derivative 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b)	**Method 1:** The particle comes to rest when $\frac{dx}{dt}=0$. $0=8\cos\left(2t-\frac{\pi}{2}\right)$ $\cos\left(2t-\frac{\pi}{2}\right)=0$ $2t-\frac{\pi}{2}=\frac{\pi}{2},\ \frac{3\pi}{2},\ \ldots$ $2t=\pi,\ 2\pi,\ \ldots$ $t=\frac{\pi}{2},\ \pi,\ \ldots$ Hence, the particle will next come to rest at $t=\frac{\pi}{2}$ seconds. **Method 2:** When $t=\frac{\pi}{2}$: $\frac{dx}{dt}=8\cos\left(2\times\frac{\pi}{2}-\frac{\pi}{2}\right)$ $=0\text{ m s}^{-1}$ $x=4\sin\left(2\times\frac{\pi}{2}-\frac{\pi}{2}\right)+8$ $=12\text{ m}$ Therefore, at $t=\frac{\pi}{2}$, the particle is at rest at 12 m to the right of the origin.	MA–C1 Introduction to Differentiation MA11–8 Bands 3–4 • Gives the correct solution . . . . 2 • Attempts to solve the trigonometric equation $\cos\left(2t-\frac{\pi}{2}\right)=0$ 1
(c)	For acceleration: $\frac{d^2x}{dt^2}=-16\sin\left(2t-\frac{\pi}{2}\right)$ When $t=\frac{\pi}{2}$: $\frac{d^2x}{dt^2}=-16\sin\left(2\times\frac{\pi}{2}-\frac{\pi}{2}\right)$ $=-16\sin\left(\frac{\pi}{2}\right)$ $=-16$ Therefore, the particle will move towards the left after being stationary at $t=\frac{\pi}{2}$.	MA–C1 Introduction to Differentiation MA11–8 Band 4 • Gives the correct solution and description of the motion of the particle . . . . 2 • Correctly finds the formula for acceleration 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 27	
(a) Since $\int_{-\infty}^{\infty} f(t)\,dt = 1$: $\int_1^{14} \frac{k}{2t-1}\,dt = 1$ $\frac{k}{2}\int_1^{14} \frac{2}{2t-1}\,dt = 1$ $\frac{k}{2}\left[\ln\lvert 2t-1\rvert\right]_1^{14} = 1$ $\frac{k}{2}\left(\ln 27 - \ln 1\right) = 1$ $\frac{k}{2}(\ln 27) = 1$ $k = \frac{2}{\ln 27}$	MA–S3 Random Variables MA12–8 Band 4 • Gives the correct solution . . . . 2 • Correctly finds the anti-derivative 1
(b) 	MA–S3 Random Variables MA12–8 Bands 4–5 • Sketches the correct graph with correct shape and points clearly labelled. . . . . 2 • Sketches the correct shape. 1
(c) Let T be the time after symptoms of the virus first appear. $\int_1^{T} \frac{2}{\ln 27} \times \frac{1}{2t-1}\,dt = \frac{3}{4}$ $\frac{2}{\ln 27} \times \frac{1}{2}\int_1^{T} \frac{2}{2t-1}\,dt = \frac{3}{4}$ $\frac{1}{\ln 27}\left[\ln\lvert 2t-1\rvert\right]_1^{T} = \frac{3}{4}$ $\ln\lvert 2T-1\rvert = \frac{3\ln 27}{4}$ $2T - 1 = e^{\frac{3\ln 27}{4}}$ $T = \frac{\left(e^{\frac{3\ln 27}{4}}\right) + 1}{2}$ $= 6.422\ldots$ ≈ 7 days	MA–S3 Random Variables MA12–10 Bands 5–6 • Gives the correct solution . . . . 2 • Correctly finds the anti-derivative and arrives at the expression $\frac{1}{\ln 27}\left[\ln\lvert 2t-1\rvert\right]_1^{T} = \frac{3}{4}$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
(a) As the initial amount of substance A is 200 grams, the time taken to decrease to half its original value is calculated as follows. Let $M_A = 100$. $100 = 200e^{-0.05t}$ $\frac{1}{2} = e^{-0.05t}$ $\ln\left(\frac{1}{2}\right) = -0.05t$ $\ln 1 - \ln 2 = -0.05t$ $\ln 2 = 0.05t$ $t = \frac{\ln 2}{0.05}$ $= 13.86\ldots$ ≈ 14 minutes Therefore, it will decrease to half its original value in 14 minutes.	MA–E1 Exponential and Logarithmic Functions MA11–8 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly arrives at the expression $100 = 200e^{-0.05t}$ and attempts to solve for t 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) The rate of change of both substances: $\frac{dM_A}{dt} = -0.05 \times 200e^{-0.05t}$ $= -10e^{-0.05t}$ $\frac{dM_B}{dt} = 400 \times \ln 3 \times -0.12 \times 3^{-0.12t}$ $= -48\ln 3 \times 3^{-0.12t}$ Equate the two rates: $-10e^{-0.05t} = -48\ln 3 \times 3^{-0.12t}$ $\frac{-10}{-48\ln 3} = \frac{3^{-0.12t}}{e^{-0.05t}}$ $= \frac{e^{\ln\left(3^{-0.12t}\right)}}{e^{-0.05t}}$ $= \frac{e^{(-0.12\ln 3)t}}{e^{-0.05t}}$ $= e^{(-0.12\ln 3 + 0.05)t}$ $0.1895\ldots = e^{-0.0818t}$ $\ln(0.1895)\ldots = -0.0818t$ $t = \frac{\ln 0.1895\ldots}{-0.0818\ldots}$ $= 20.317\ldots$ minutes ≈ 20 minutes 19 seconds Therefore, both substances decay at the same rate at 20 minutes and 19 seconds.	MA–E1 Exponential and Logarithmic Functions MA11–8 Bands 5–6 • Gives the correct solution . . . . 4 • Writes an expression using the same base . . . . 3 • Correctly finds the rates of decay for both substances AND attempts to solve the equation $-10e^{-0.05t} = -48\ln 3 \times 3^{-0.12t}$ . . . . 2 • Find the rate of change for substance *A* OR substance *B* 1

<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>Question 29</td><td></td></tr>
<tr><td>

For the point A:

$$\begin{aligned}4-3x^2&=-x\\0&=3x^2-x-4\\0&=(3x-4)(x+1)\\x&=\frac{4}{3}\text{ or }-1\end{aligned}$$

Therefore, $x=-1$ according to the diagram.

When $x=-1$:

$$\begin{aligned}y&=4-3(-1)^2\\&=1\end{aligned}$$

Therefore, $A(-1,\ 1)$.

Due to the symmetry of $y=4-3x^2$, $C(1,\ 1)$.

$$\begin{aligned}A_{ABC}&=\int_{-1}^{1}4-3x^2dx-A_{\text{rectangle}}\\&=\left[4x-x^3\right]_{-1}^{1}-2\\&=(4-1)-(-4+1)-2\\&=4\\A_{\text{logo}}&=4\times A_{ABC}+2\times A_{\text{rectangle}}\\&=20\text{ units}^2\end{aligned}$$

</td><td>

MA–C4 Integral Calculus

MA12–7 Bands 5–6

- Gives the correct solution 4

- Correctly uses the points $A(-1,\ 1)$ and $C(1,\ 1)$ to find the area of ABC OR equivalent merit. 3

- Correctly uses the points $A(-1,\ 1)$ and $C(1,\ 1)$ to develop an integral that represents the area of ABC OR equivalent merit. 2

- Develops an equation to show either point $A(-1,\ 1)$ OR point $C(1,\ 1)$ 1

</td></tr>
<tr><td>Question 30</td><td></td></tr>
<tr><td>

(a)

$$\begin{aligned}A_1&=1000(1.04)\\A_2&=\left(A_1+1000\right)(1.04)\\&=\left(1000(1.04)+1000\right)(1.04)\\&=1000(1.04)^2+1000(1.04)\\A_3&=\left(A_2+1000\right)(1.04)\\&=\left(1000(1.04)^2+1000(1.04)+1000\right)(1.04)\\&=1000(1.04)^3+1000(1.04)^2+1000(1.04)\\&=1000(1.04)\left(1+1.04+1.04^2\right)\end{aligned}$$

</td><td>

MA–M1 Modelling Financial Situations

MA12–4 Band 4

- Gives the correct solution 1

</td></tr>
</table>

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $A_{18} = 1000(1.04)^{18} + 1000(1.04)^{17} + \ldots + 1000(1.04)^2 + 1000(1.04)$ $= 1000(1.04)\left(1.04^{17} + 1.04^{16} + \ldots + 1.04 + 1\right)$ $a = 1,\ r = 1.04,\ n = 18$ $S_{18} = \dfrac{1\left(1.04^{18} - 1\right)}{1.04 - 1}$ $= \dfrac{1.04^{18} - 1}{0.04}$ $A_{18} = \dfrac{1000(1.04)\left(1.04^{18} - 1\right)}{0.04}$ $= 26\,000\left(1.04^{18} - 1\right)$	MA–M1 Modelling Financial Situations MA12–4 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly finds an unsimplified expression for A_{18} 1
(c) $A_{20} = \left[26\,000\left(1.04^{18} - 1\right)(1.04) - M\right](1.04) - M$ $= 26\,000\left(1.04^{18} - 1\right)(1.04)^2 - M(1.04) - M$ $= 26\,000\left(1.04^{18} - 1\right)(1.04)^2 - M(1 + 1.04)$ $A_{21} = 26\,000\left(1.04^{18} - 1\right)(1.04)^3 - M\left(1 + 1.04 + 1.04^2\right)$ $A_{22} = 26\,000\left(1.04^{18} - 1\right)(1.04)^4 - M\left(1 + 1.04 + 1.04^2 + 1.04^3\right)$ $= 26\,000\left(1.04^{18} - 1\right)(1.04)^4 - M\left[\dfrac{(1)\left(1.04^4 - 1\right)}{1.04 - 1}\right]$ $= 26\,000\left(1.04^{18} - 1\right)(1.04)^4 - 25M\left(1.04^4 - 1\right)$ When $A_{22} = 0$: $26\,000\left(1.04^{18} - 1\right)(1.04)^4 - 25M\left(1.04^4 - 1\right) = 0$ $26\,000\left(1.04^{18} - 1\right)(1.04)^4 = 25M\left(1.04^4 - 1\right)$ $\dfrac{26\,000\left(1.04^{18} - 1\right)(1.04)^4}{25\left(1.04^4 - 1\right)} = M$ $M = \$7347.66$	MA–M1 Modelling Financial Situations MA12–4 Band 6 • Gives the correct solution . . . . 5 • Correctly uses $A_{22} = 0$ and attempts to solve . . . . 4 • Correctly uses the pattern to write down an expression for A_{22} . . . . 3 • Correctly finds an expression for A_{21} . . . . 2 • Correctly finds an expression for A_{20} 1
Question 31	
(a) (i) $4\cos 4x = \dfrac{1}{2}\sin 4x$ $8\cos 4x = \sin 4x$ $8 = \tan 4x$ $\therefore \tan 4x = 8$	MA–T2 Trigonometric Functions and Identities MA12–4 Bands 3–4 • Gives the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\tan 4x = 8$ in $[0, 4\pi]$ $4x = \tan^{-1} 8,\ (\pi + \tan^{-1} 8),\ (2\pi + \tan^{-1} 8)$ $x = \frac{1}{4}\tan^{-1} 8,\ \frac{1}{4}(\pi + \tan^{-1} 8),$ $\frac{1}{4}(2\pi + \tan^{-1} 8)$ Therefore, solutions in the domain [0, π] are: $x_1 = \frac{1}{4}\tan^{-1} 8$ $x_2 = \frac{1}{4}(\pi + \tan^{-1} 8)$	MA–T2 Trigonometric Functions and Identities MA11–4 Bands 4–5 • Gives the correct solutions . . . . . . 2 • Correctly shows ONE solution 1
(b) (i) $y = 10e^{-\frac{1}{2}x} \sin 4x$ $y' = 10\left[\sin 4x \times -\frac{1}{2}e^{-\frac{1}{2}x} + e^{-\frac{1}{2}x} \times 4\cos 4x\right]$ $= 10e^{-\frac{1}{2}x}\left(-\frac{1}{2}\sin 4x + 4\cos 4x\right)$ For stationary points, $y' = 0$: $10e^{-\frac{1}{2}x}\left(-\frac{1}{2}\sin 4x + 4\cos 4x\right) = 0$ $e^{-\frac{1}{2}x} = 0$ or $-\frac{1}{2}\sin 4x + 4\cos 4x = 0$ There are no real solutions for $e^{-\frac{1}{2}x} = 0$, as $e^{-\frac{1}{2}x} > 0$ for all real x. Therefore: $x_1 = \frac{1}{4}\tan^{-1} 8$ $x_2 = \frac{1}{4}(\pi + \tan^{-1} 8)$ *Note: Consequential on answer to part (a)(ii).*	MA–C3 Applications of Differentiation MA12–3 Bands 5–6 • Gives the correct solution . . . . 2 • Finds the derivative of y 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) When $X_1 = \frac{1}{4}\tan^{-1} 8$: $Y_1 = 10e^{-\frac{1}{2}\left(\frac{1}{4}\tan^{-1}8\right)}\sin\left[4\left(\frac{1}{4}\tan^{-1}8\right)\right]$ $= 10e^{-\frac{1}{8}\tan^{-1}8}\sin\left(\tan^{-1}8\right)$ When $X_2 = \frac{1}{4}\left(\pi + \tan^{-1}8\right)$: $Y_2 = 10e^{-\frac{1}{2}\left[\frac{1}{4}\left(\pi+\tan^{-1}8\right)\right]}\sin\left[4\left(\frac{1}{4}\left(\pi+\tan^{-1}8\right)\right)\right]$ $= 10e^{-\frac{1}{8}\left(\pi+\tan^{-1}8\right)}\sin\left(\pi+\tan^{-1}8\right)$ $= 10e^{-\frac{1}{8}\left(\pi+\tan^{-1}8\right)}\times -\sin\left(\tan^{-1}8\right)$ $= -10e^{-\frac{1}{8}\left(\pi+\tan^{-1}8\right)}\sin\left(\tan^{-1}8\right)$ Common ratio: $r = \frac{Y_2}{Y_1}$ $= \frac{-\cancel{10}e^{-\frac{1}{8}\left(\pi+\tan^{-1}8\right)}\cancel{\sin\left(\tan^{-1}8\right)}}{\cancel{10}e^{-\frac{1}{8}\tan^{-1}8}\cancel{\sin\left(\tan^{-1}8\right)}}$ $= -e^{-\frac{1}{8}\left(\pi+\tan^{-1}8\right)-\left(-\frac{1}{8}\tan^{-1}8\right)}$ $= -e^{-\frac{1}{8}\pi-\cancel{\frac{1}{8}\tan^{-1}8}+\cancel{\frac{1}{8}\tan^{-1}8}}$ $= -e^{-\frac{1}{8}\pi}$	MA–M1 Modelling Financial Situations MA–C3 Applications of Differentiation MA12–4, MA12–10 Band 6 • Gives the correct solution with $r = e^{-\frac{1}{8}\pi}$ .... 3 • Correctly finds an expression for Y_1 and Y_2 .... 2 • Correctly finds an expression for Y_1 1