Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5–8)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Given $P = 100 + 300e^{-0.2t}$, what value does P approach as t approaches infinity?

A. 100

B. 200

C. 300

D. 400

2 How many three-digit odd numbers can be formed by using the digits 1, 2, 3, 4 and 5 if repetitions are NOT allowed?

A. 24

B. 36

C. 48

D. 60

3 Which of the following expressions is equivalent to $2\sin\left(x + \frac{\pi}{3}\right)$?

A. $\sqrt{3}\sin x + \cos x$

B. $\sqrt{3}\sin x - \cos x$

C. $\sin x + \sqrt{3}\cos x$

. $\sin x - \sqrt{3}\cos x$

4 The function $f(x)$ is defined as $f(x) = 3x^3 + 4$.

Which of the following expressions is equal to $f^{-1}(x)$?

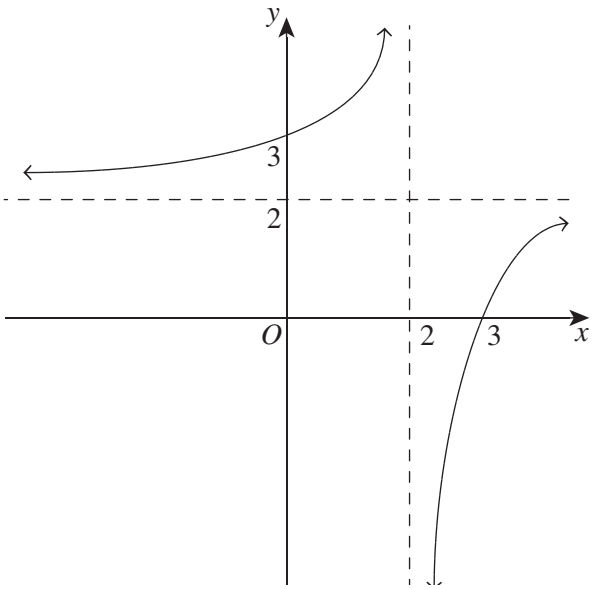
A. $\frac{1}{3x^3} + 4$

B. $\frac{1}{3x^3 + 4}$

C. $\frac{\sqrt[3]{x+3}}{4}$

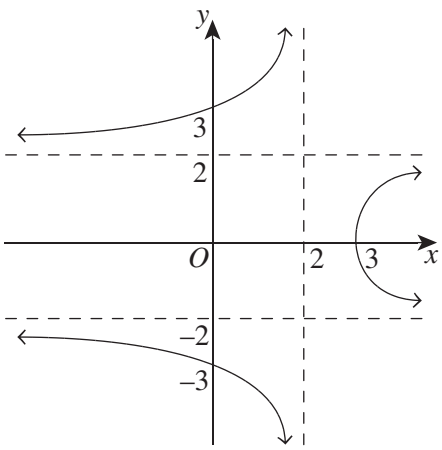
. $\sqrt[3]{\frac{x-4}{3}}$

5 The graph of $y = f(x)$ is given below.

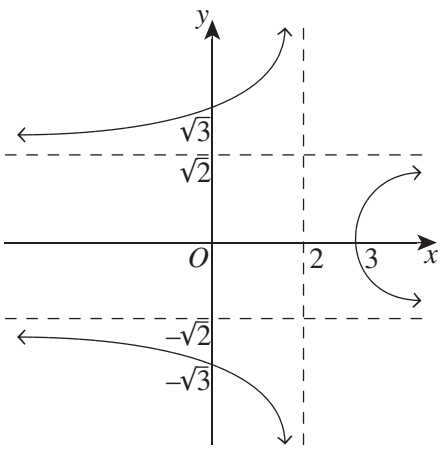


The graph of $y^2 = f(x)$ is

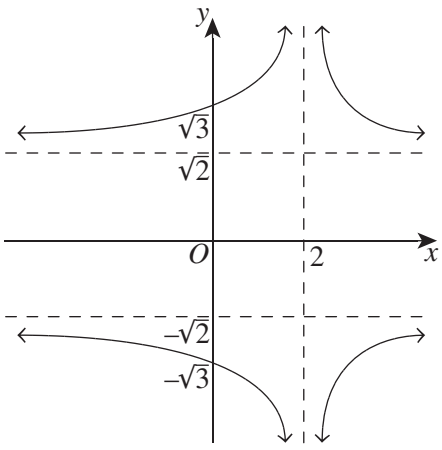
A.



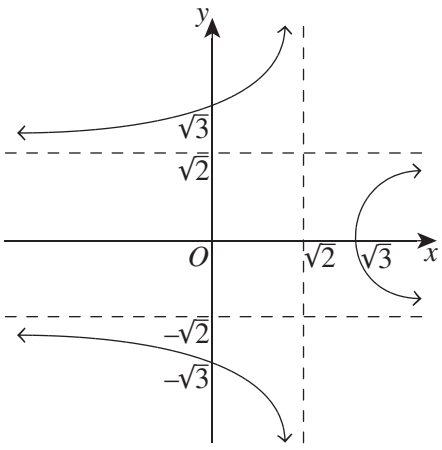
B.



C.



D.



6 If $t = \tan\left(\frac{\theta}{2}\right)$, the correct expression for $\frac{\text{cosec}^2\theta}{1+\tan^2\theta}$ is

A. $\frac{(1+t^2)^2}{(1-t^2)^2}$

B. $\frac{1+t^2}{(1-t^2)^2}$

C. $\frac{4t^2}{(1-t^2)^2}$

D. $\frac{(1-t^2)^2}{4t^2}$

7 The parametric equations of a function are $x = \frac{t}{1+t}, y = \frac{t}{1-t}$, where $t \neq \pm 1$.

The Cartesian equation is

A. $x - 2xy + y = 0$

B. $x - 2xy - y = 0$

C. $x + 2xy - y = 0$

D. $x + 2xy + y = 0$

8 Which of the following is the range of the function $f(x) = \cos^{-1}x + \sin^{-1}x + \tan^{-1}x$?

A. $\frac{\pi}{4} \leq y \leq \frac{3\pi}{4}$

B. $\frac{\pi}{4} < y < \frac{3\pi}{4}$

C. $0 \leq y \leq \pi$

D. $0 < y < \pi$

9 What is the least number of distinct integers that can be chosen from the sequence 1, 3, 5, 7, …, 97, 99 so that it is guaranteed that two of them will have a sum of 102?

A. 24

B. 25

C. 26

D. 27

10 By considering the binomial expansion of $(1+x)^{10}$, what is the value of

$3\binom{10}{1} + 3^2\binom{10}{2} + 3^3\binom{10}{3} + \ldots + 3^{10}\binom{10}{10}$?

A. 2^{10}

B. 4^{10}

C. $2^{10} - 1$

D. $4^{10} - 1$

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Find the linear factors of $x^3 + 3x^2 - 13x - 15$. **2**

(ii) Hence, solve $x^3 + 3x^2 - 13x - 15 > 0$. **2**

(b) Solve $\dfrac{2x-5}{3x-2} \leq 2$. **3**

(c) Find the value of k if ${}^8C_k = 2 \times {}^7C_k$. **2**

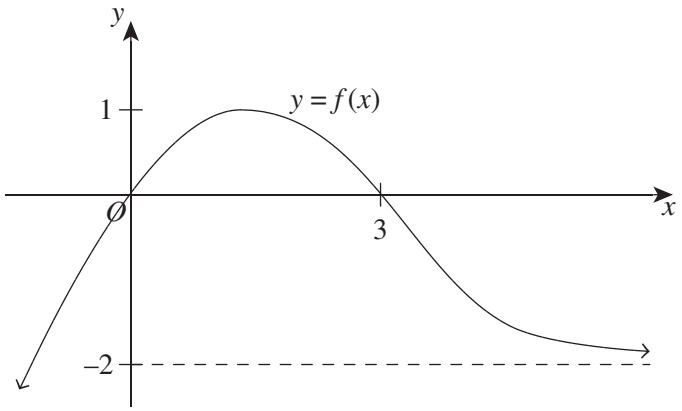
(d) Find the coefficient of x^4 in the expansion of $\left(2 + \dfrac{x}{4}\right)^6$. **2**

(e) Prove that $\tan\theta \tan\dfrac{\theta}{2} = \sec\theta - 1$. **2**

(f) Write $\sin\left(2\cos^{-1}\left(-\dfrac{2}{5}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The graph $y = f(x)$ is shown.



(i) Sketch the graph of $y = \dfrac{1}{f(x)}$. **2**

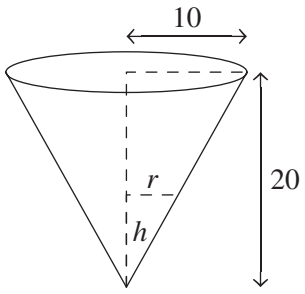
(ii) Sketch the graph of $y = \sqrt{f|x|}$. **2**

(b) (i) Sketch the graph of $f(x) = |x+2| + |x-2|$. **2**

(ii) On the same set of axes, sketch $h(x) = 2x + 6$. **1**

(iii) Hence, or otherwise, solve $f(x) \geq h(x)$. **1**

(c) Sand is being poured into a right conical flask of radius, r, 10 cm and height, h, 20 cm at a rate of 1 cm^3/s. **3**



How fast is the sand level rising when the depth of the sand is 2 cm?

(d) Find the exact value of x so that $\tan^{-1} x = \tan^{-1}\left(\dfrac{1}{2}\right) - \tan^{-1}\left(\dfrac{1}{3}\right)$. **2**

(e) Find the number of words that can be made by all of the letters in the word GEOMETRY so that no vowels are adjacent. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The equation $x^4 - 3x^3 - 6x^2 + ax + b = 0$ has a triple root. **3**
Find a and b, and hence all roots of this equation.

(b) Two of the roots of $4x^3 + 7x^2 + kx + 24$ are reciprocals. **3**
Find the THREE roots of this equation AND evaluate k.

(c) Consider the function $f(x) = x^2 - 6x + 10$.

(i) Explain why the inverse of $f(x)$ is NOT a function. **1**

(ii) What is the largest domain of $f(x)$ containing $x = 0$ for which $f^{-1}(x)$ exists? **1**

(iii) Find an expression for $f^{-1}(x)$ using the domain found in part (c) (ii). **2**

(iv) Find the point of intersection where $f(x) = f^{-1}(x)$. **1**

(d) A hand of 13 cards is taken out of a well-shuffled pack of 52 cards. **2**
How many different hands of 13 cards consist of at least THREE aces and THREE kings?

(e) Prove that $\sin 40° + \cos 70° = \cos 10°$. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) An object falls from a skyscraper. The rate of change of its velocity v is given by $\frac{dv}{dt} = -k(v - 100)$, where k is a constant.

(i) Show that $v = 100 - 100e^{-kt}$ is a possible equation to this differential equation. **1**

(ii) The velocity after 10 seconds is 40 m/s. **1**

Find the value of k correct to FOUR decimal places.

(iii) Find the velocity after a further 15 seconds. **1**

(iv) Find the limiting velocity. **1**

(v) Sketch the graph of v versus t. **1**

(b) State the domain and range of $f(x) = 3\cos^{-1}(5 - 2x)$ and, hence, sketch the graph of $y = f(x)$. **3**

(c) By expanding both sides of the identity $(1+x)^{15} = (1+x)^{12}(1+x)^3$, prove that **3**

$$\binom{15}{4} = \binom{12}{4} + 3\binom{12}{3} + 3\binom{12}{2} + \binom{12}{1}.$$

(d) (i) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **2**

(ii) Hence, or otherwise, show that $\sin 18° = \frac{\sqrt{5}-1}{4}$. **2**

End of paper

MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

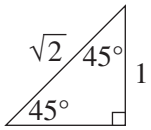
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}},\ \cos A = \frac{\text{adj}}{\text{hyp}},\ \tan A = \frac{\text{opp}}{\text{adj}}$$

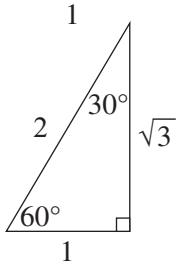
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\text{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

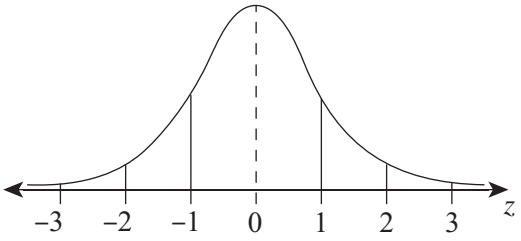
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 - 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)dx$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r\, p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1+\left[f(x)\right]^2}}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 - \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r\left(\cos\theta + i\sin\theta\right)$$
$$= re^{i\theta}$$

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

Section II Writing Booklet

Question Number

Student Name/Number: _____

Instructions

Use a separate writing booklet for each question in Section II.

Write the number of this booklet and the total number of booklets that you have used for this question (e.g. 1 of 3)

⇒ [ ] of [ ]

this booklet | number of booklets for this question

Write in black or blue pen (black is recommended).

You may ask for an extra writing booklet if you need more space.

If you have not attempted the question(s), you must still hand in a writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.

You may NOT take any writing booklets, used or unused, from the examination room.

Tick this box if you have continued this answer in another writing booklet. ☐

Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

DIRECTIONS:

Write your name in the space provided.

Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.

Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one oval** per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: _____

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I

MULTIPLE-CHOICE ANSWER SHEET

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II

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