CSSA

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAM ROOM

Centre Number

Student Number

Mathematics Extension 1

Afternoon Session
Friday, 6 August 2021

General Instructions

- Reading time-10 minutes
- Working time-2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks-70

Section I Pages 2-6

10 marks

- Attempt Questions 1-10
- Allow 15 minutes for this section

Section II Pages 7-11

60 marks

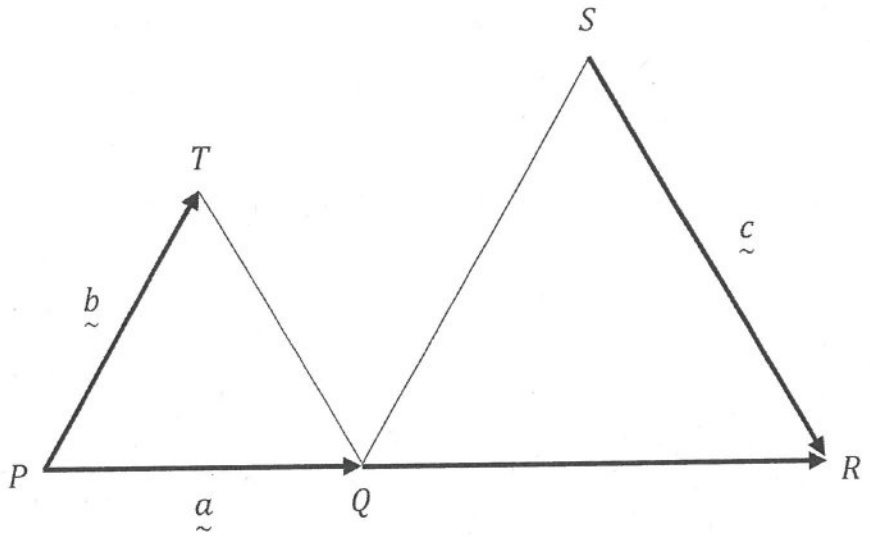
- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1-10.

1



The figure shown above consists of two equilateral triangles where P, Q, and R are collinear with $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PT} = \underset{\sim}{b}$ and $\overrightarrow{SR} = \underset{\sim}{c}$.
Which of the following statement is true?

A. $\underset{\sim}{a} = \underset{\sim}{b}$

B. $\overrightarrow{QR} = \underset{\sim}{c}$

C. $\overrightarrow{SQ} = \underset{\sim}{b}$

D. $|\underset{\sim}{b}| = |\underset{\sim}{a}|$

2 What is the Cartesian equation whose parametric equations are given by: $x = 4\sin\theta - 1$ and $y = 3\cos\theta + 2$?

A. $\dfrac{(x-1)^2}{4} + \dfrac{(y-2)^2}{3} = 1$

B. $\dfrac{(x+1)^2}{4} + \dfrac{(y-2)^2}{3} = 1$

C. $\dfrac{(x-1)^2}{16} + \dfrac{(y+2)^2}{9} = 1$

D. $\dfrac{(x+1)^2}{16} + \dfrac{(y-2)^2}{9} = 1$

3 The polynomial $f(x) = 2x^2 + kx + 4$ can be expressed as $f(x) = (x-2)g(x) + 6$. Which of the following is the correct expression for $g(x)$?

A. $2x - 1$

B. $2x + 1$

C. $2x - 3$

D. $2x + 3$

4 Which of the following expressions is correct?

A. $\sin^{-1} x = \cos^{-1}(-x) - \dfrac{\pi}{2}$

B. $\sin^{-1} x = -\cos^{-1} x - \dfrac{\pi}{2}$

C. $\sin^{-1} x = -\cos^{-1}(-x) - \dfrac{\pi}{2}$

D. $\sin^{-1} x = -\cos^{-1}(-x) + \dfrac{\pi}{2}$

5 Which of the following functions has a domain $[1, 5]$ and range $[1, 4\pi + 1]$?

A. $f(x) = 2\sin^{-1}\left(\frac{x-3}{2}\right)+1$

B. $f(x) = 4\cos^{-1}\left(\frac{x-3}{2}\right)+1$

C. $f(x) = 4\sin^{-1}\left(\frac{x-3}{2}\right)+1$

D. $f(x) = 2\cos^{-1}\left(\frac{x-3}{2}\right)+1$

6 The temperature, T, in degrees Celsius, of a metal bar being heated after t hours is defined as $T = 50 + Be^{kt}$, where B and k are constants. The metal bar's temperature was measured to be $1800°$ C after 2 hours of being heated.
After a further 2 hours its temperature was $2500°$ C. What would the approximate time be when it reaches a temperature of $2200°$ C?

A. 3 hours and 42 minutes

B. 3 hours and 13 minutes

C. 2 hours and 42 minutes

D. 2 hours and 13 minutes

7 Given the points $A\begin{pmatrix}2\\3\end{pmatrix}$, $B\begin{pmatrix}1\\4\end{pmatrix}$ and $C\begin{pmatrix}3\\q\end{pmatrix}$, it is also known that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$.
What is the value of q?

A. -6

B. -2

C. 2

D. 6

8 A Bernoulli variable, X, has a value of p such that $E(X) = 3\text{Var}(X)$.
Given that $p \neq 0$, what is the value of p?

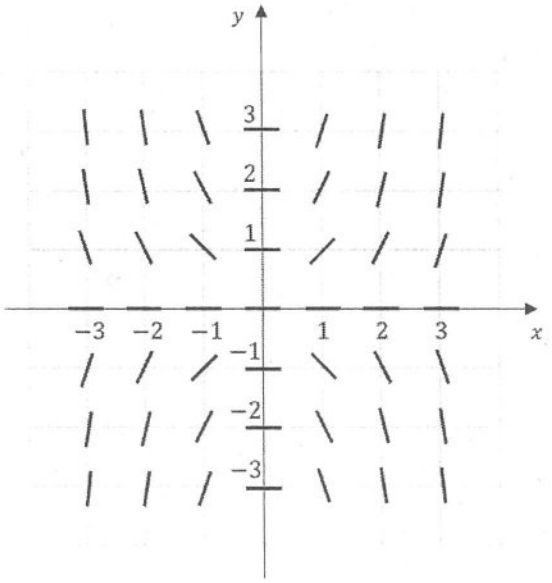
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

9 Which of the following differential equations is represented by the slope field below?



A. $y \times \frac{dy}{dx} = x$

B. $x \times \frac{dy}{dx} = y$

C. $\frac{dy}{dx} + xy = 0$

D. $\frac{dy}{dx} - xy = 0$

10 There are 26 cards in a bag, each has a different letter of the alphabet written on them.

A game consists of drawing cards one at a time, without replacement, until two consecutive letters of the alphabet have been drawn. A and Z are not consecutive letters.

For example, if B is drawn first and M is drawn second, if the third card is either A, C, L or N the game would stop there as A and B, or B and C, or L and M, or M and N form a consecutive pair of letters. There would be 23 letters left in the bag.

What is the least number of cards that can be left in the bag at the end of the game?

A. 11

B. 12

C. 13

D. 14

End of Section I

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

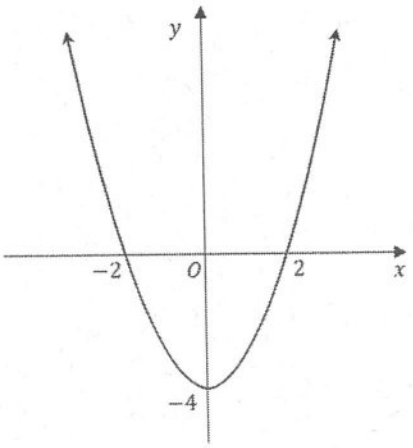
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Your responses in Questions 11-14 should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{3}{x-2} < 4$. **2**

(b) The graph below shows $y = f(x)$ where $f(x) = x^2 - 4$. Sketch the following curves on separate graphs, each at least one-third of a page in size.



(i) $y = -f(x)$. **1**

(ii) $y = |f(x)|$. **1**

(c) How many arrangements of the 8 letters of the word PROBLEMS are possible if the vowels cannot be adjacent to each other? **2**

(d) The polynomial $P(x) = ax^3 + bx^2 + c$ has a double zero at $x = 2$ and has remainder -64 when divided by $x + 2$. Find a, b and c. **3**

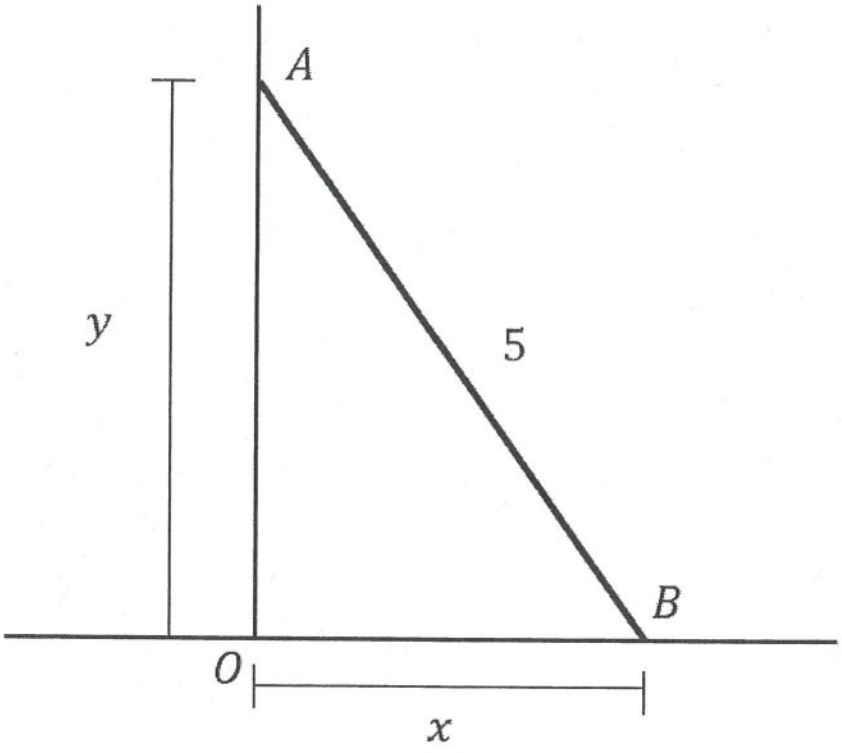
(e) Given that $\cos x - \sin x = \dfrac{1}{3}$ and $0 < x < \dfrac{\pi}{4}$, find the exact value of $\cot 2x$. **3**

(f) Find the value of n if $\int_0^{\frac{2}{3}} \dfrac{dx}{4+9x^2} = n\pi$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Using the substitution $u = 1 + e^x$ or otherwise, find $\int \frac{e^{3x}}{1+e^x} dx$. **3**

(b) A ladder, AB, of length 5 metres, is leaning against a vertical wall OA (y metres), with its foot B, on horizontal ground OB (x metres). The foot of the ladder begins to slide along the ground away from the wall at a constant speed of 1 metre per second. **3**



Find the rate at which the top of the ladder A is moving down the wall at the time when the top of the ladder is 4 metres above the ground.

(c) Prove by mathematical induction that $4^n + 14$ is a multiple of 6 for $n \geq 1$. **3**

Question 12 is continued on page 9

Question 12 continued

(d) The probability that a player hits the bullseye on any one throw of a dart is 70%. Let X be a binomial random variable representing the number of times the player hits the bullseye in 100 consecutive throws.

(i) Find the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **2**

(ii) Find the probability that they hit the bullseye exactly 70 times from the 100 throws, correct to 3 significant figures. **2**

Let $\hat{p}$ represent the average number of times they hit the bullseye per throw from the 100 consecutive throws.

(iii) Using a normal approximation to $\hat{p}$, or otherwise, find an approximation to 2 decimal places for the probability that the average number of times they hit the bullseye per throw is less than 65%. **2**

The table below shows a section of the normal distribution table $P(Z < z)$.

z	− 0.00	− 0.01	− 0.02	− 0.03	− 0.04	− 0.05	− 0.06	− 0.07	− 0.08	− 0.09
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
-0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
z	− 0.00	− 0.01	− 0.02	− 0.03	− 0.04	− 0.05	− 0.06	− 0.07	− 0.08	− 0.09

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

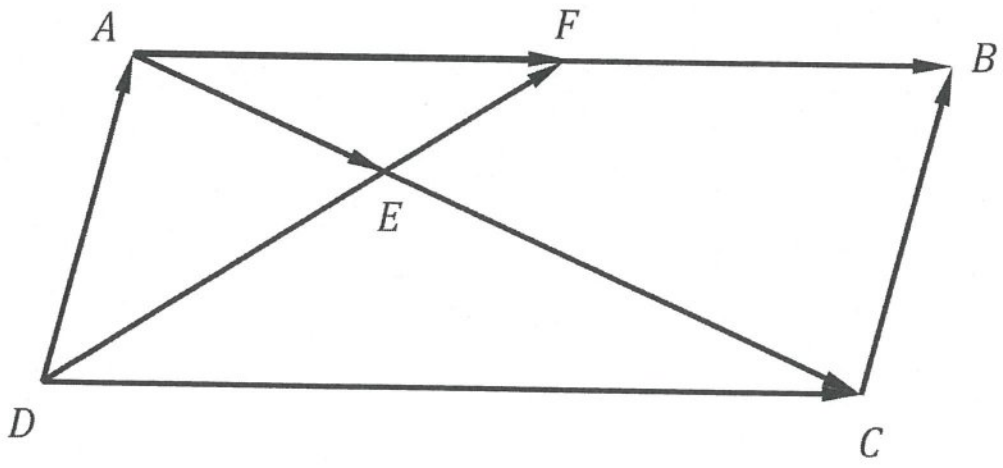
(a) Given the points $A\begin{pmatrix}1\\-1\end{pmatrix}$ and $B\begin{pmatrix}1\\4\end{pmatrix}$, show that $\angle AOB = 121°$ to the nearest degree, where O is the origin. **2**

(b) Find the volume generated when the area bounded by $y = 1 - \cos x$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$ and the x axis is rotated about the x axis. **4**

(c) Solve $3\sin x - 4\cos x = 2.5$ for $[-\pi, \pi]$. **4**

(d) Find the equation of a curve, $y = f(x)$ given that, $f'(x) = \frac{2}{4+x^2}$ and the curve passes through the point $\left(2, \frac{\pi}{2}\right)$. **2**

(e) $ABCD$ is a parallelogram and F is a point on AB such that AC and DF intersect at E. If $\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AC}$, prove that $\overrightarrow{AF} = \frac{2}{3}\overrightarrow{DC}$. Let $\overrightarrow{DA} = \underset{\sim}{u}$ and $\overrightarrow{DC} = \underset{\sim}{v}$ **3**



Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Using the expansion $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$, **2**

show that $2^{n-1} = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots + \binom{n}{n}$ for odd n.

(b) Find the term independent of x in the expansion of **3**

$$\left(5 + \frac{4}{x^3}\right)\left(2x + \frac{3}{x^2}\right)^{15}.$$

(c) A ball is projected upwards from the origin on a horizontal surface with an initial velocity of $\underset{\sim}{v}(0) = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$ m/s.

Assume $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -g \end{pmatrix}$, where $g = 10$ m/s^2, and that air resistance is negligible.

(i) Prove that the velocity t seconds after projection is given by **2**

$$\underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 20\sqrt{3} - 10t \end{pmatrix}.$$

(ii) Find how long the ball is in the air before impact with the horizontal surface. **2**

(iii) Prove that the maximum magnitude of $\underset{\sim}{v}(t)$ at any time during flight is twice its minimum magnitude. **2**

(d) (i) Arrythmia is an abnormal heart condition where the heart beat is irregular. The amplitude, A, of a patient's heart beat varies with time, t as follows **1**

$$\frac{dA}{dt} = \frac{\tan^3 t}{A\cos^4 t}$$

Show that $\dfrac{\tan^3 t}{\cos^4 t} = \sec^2 t(\tan^5 t + \tan^3 t)$.

(ii) Given that $A\left(\dfrac{\pi}{4}\right) = 1$, find the equation of A as a function of time t. **3**

End of examination

02 September 2021

URGENT AND CONFIDENTIAL

MEMORANDUM TO PRINCIPALS / CURRICULUM COORDINATORS / HEADS OF DEPARTMENTS

RE: ERRATA FOR THE 2021 CSSA TRIAL HSC EXAMINATIONS

You have received this Errata via email as you are the ONLY CSSA main contact person for your school/college in the CSSA website, based on entered data by your school/college.

Please ensure that the persons in your school/college responsible for the relevant department receives the following urgent and confidential information today.

Examination: **MATHEMATICS EXTENSION 1**
Date: **TUESDAY 01 SEPTEMBER 2021**
Session: **Afternoon PM**

It is with considerable regret that it is necessary to advise schools/colleges of an error in the Sample Answer of the Marking Guidelines that may impact on the marking of Question 12 (d) (iii).

Question 12 (d) (iii) page 12
should read

Sample Answer:

$$\sigma = \sqrt{\frac{0.7 \times 0.3}{100}} = \sqrt{0.0021}$$

$$z = \frac{0.65 - 0.7}{\sqrt{0.0021}} = -1.09 \text{ (2 dp)}$$

$P(z < -1.09) = 0.1379$ (from z tables)

The probability that $\hat{p} < 65\%$ is 0.14.

Also accept calculations using 0.645 (continuity correction):

$$\sigma = \sqrt{\frac{0.7 \times 0.3}{100}} = \sqrt{0.0021}$$

$$z = \frac{0.645 - 0.7}{\sqrt{0.0021}} = -1.20 \text{ (2 dp)}$$

$P(z < -1.20) = 0.1151$ (from z tables)

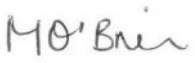
The probability that $\hat{p} < 65\%$ is 0.12.

Note: Published CSSA Trial HSC Examination Marking Guidelines (marking criteria) are accurate however the Sample Answer requires updating to accommodate this Errata.

At this time, schools/colleges are reminded:

- of the importance in maintaining the security of the CSSA Trial HSC Examinations and associated documents, and Marking Guidelines for the integrity of the CSSA Trial HSC Examination program
- to **collect and retain CSSA Trial HSC Examinations and associated documents and with students' scripts** until the end of the nominated CSSA Trial HSC Examination security period, which is **8am, Monday 6 September 2021**.
- CSSA Trial HSC Examinations and associated documents and students' scripts can be given back **from 8.01am, Monday 6 September 2021**.

Yours sincerely



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