2021
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

CSSA

MARKING GUIDELINES

Mathematics Extension I

Section I
10 marks
Multiple Choice Answer Key

Question	Answer
1	D
2	D
3	B
4	A
5	B
6	B
7	D
8	C
9	D
10	B

Question 1 (1 mark)
Outcomes Assessed: ME-V1.1/ME12-2
Targeted Performance Bands: E2-E3

Solution	Mark
$\underset{\sim}{a} \neq \underset{\sim}{b}$ so not (A) $\overrightarrow{QR} \neq \underset{\sim}{c}$ so not (B) $\overrightarrow{SQ} \neq \underset{\sim}{b}$ so not (C) $\lvert\underset{\sim}{b}\rvert = \lvert\underset{\sim}{a}\rvert$ is true Hence (D)	1

Question 2 (1 mark)
Outcomes Assessed: ME-F1.4/ME11-1
Targeted Performance Bands: E2-E3

Solution	Mark
$x = 4\sin\theta - 1 \Rightarrow x + 1 = 4\sin\theta$ $$\sin\theta = \frac{x+1}{4}$$ $y = 3\cos\theta + 2 \Rightarrow y - 2 = 3\cos\theta$ $$\cos\theta = \frac{y-2}{3}$$ Using $\sin^2\theta + \cos^2\theta = 1$ $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{9} = 1$ Hence (D)	1

Question 3 (1 mark)
Outcomes Assessed: ME-F2.1/ME11-2
Targeted Performance Bands: E2-E3

Solution	Mark
$f(x) = (x-2)g(x) + 6$ then $f(2) = 6$ $f(2) = 2\times 2^2 + k\times 2 + 4 = 6$ $12 + 2k = 6$ $2k = -6$ $k = -3$ $2x^2 - 3x + 4 = (x-2)g(x) + 6$ $2x^2 - 3x - 2 = (x-2)g(x)$ $(x-2)(2x+1) = (x-2)g(x)$ $g(x) = (2x+1)$ Hence (B)	1

Question 4 (1 mark)
Outcomes Assessed: ME-T1/ME11-3
Targeted Performance Bands: E2-E3

Solution	Mark
Since $\cos^{-1}(-x) = \pi - \cos^{-1} x$ and $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $\Rightarrow \cos^{-1} x = \pi - \cos^{-1}(-x)$ $\Rightarrow \frac{\pi}{2} - \sin^{-1} x = \pi - \cos^{-1}(-x)$ $\therefore \sin^{-1} x = \cos^{-1}(-x) - \frac{\pi}{2}$ Hence (A)	1

Question 5 (1 mark)
Outcomes Assessed: ME-T1/ME11-3
Targeted Performance Bands: E2-E3

Solution	Mark
All options have the correct domain of $[1,5]$. The correct range can be found by dilating $y = \cos^{-1}\left(\frac{x-3}{2}\right)$ vertically by a factor of 4 and translating the curve up 1 unit, so $y = 4\cos^{-1}\left(\frac{x-3}{2}\right) + 1$ Hence (B)	1

Question 6 (1 mark)
Outcomes Assessed: ME-C1.2/ME11-4
Targeted Performance Bands: E3-E4

Solution	Mark
$T = 50 + Be^{kt}$ $t = 2, T = 1800 \Rightarrow 1800 = 50 + Be^{2k}$ $1750 = Be^{2k} \quad (1)$ $t = 4, T = 2500 \Rightarrow 2500 = 50 + Be^{4k}$ $2450 = Be^{4k} \quad (2)$ $(2) \div (1) \qquad e^{2k} = \frac{2450}{1750} = \frac{7}{5}$ $\Rightarrow \quad k = \frac{1}{2}\ln\left(\frac{7}{5}\right),\ B = 1250$ $T = 2200 \quad \Rightarrow 2200 = 50 + 1250e^{\frac{1}{2}\ln\left(\frac{7}{5}\right)t}$ $\therefore\ t \approx 3.2236$ hours $\approx$ 3 hours 13 minutes Hence (B)	1

Question 7 (1 mark)
Outcomes Assessed: ME-V1.2/ME12-2
Targeted Performance Bands: E3-E4

Solution	Mark
$\overrightarrow{AB}.\overrightarrow{BC} = 0$ $\begin{pmatrix} 1-2 \\ 4-3 \end{pmatrix}.\begin{pmatrix} 3-1 \\ q-4 \end{pmatrix} = 0$ $\begin{pmatrix} -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ q-4 \end{pmatrix} = 0$ $-2 + q - 4 = 0$ $q - 6 = 0$ $q = 6$ Hence (D)	1

Question 8 (1 mark)
Outcomes Assessed: ME-S1.1/ME12-5
Targeted Performance Bands: E3-E4

Solution	Mark
$p = 3p(1-p)$ $p = 3p - 3p^2$ $3p^2 - 2p = 0$ $p(3p-2) = 0$ $\therefore\ p = \frac{2}{3}$ since $p \neq 0$ Hence (C)	1

Question 9 (1 mark)
Outcomes Assessed: ME-C3.2/ME12-4
Targeted Performance Bands: E3-E4

Solution	Mark
Rearranging we get: (A) $\frac{dy}{dx} = \frac{x}{y}$ which would be undefined when $y = 0$ which is untrue so not (A) (B) $\frac{dy}{dx} = \frac{y}{x}$ which would be undefined when $x = 0$ which is untrue so not (B) (C) $\frac{dy}{dx} = -xy$ which would be negative in the first quadrant where $x, y > 0$ which is untrue so not (C) (D) $\frac{dy}{dx} = xy$ which would be positive in the first and third quadrants, negative in second and fourth, and zero on either axes which is true. Hence (D)	1

Question 10 (1 mark)
Outcomes Assessed: ME-A1.1/ME11-5
Targeted Performance Bands: E3-E4

Solution	Mark
Break the alphabet into 13 consecutive pairs: *AB*, *CD*, … *YZ*. There are 13 pigeonholes. There can be up to 14 cards drawn before a consecutive pair is made, leaving 12 cards in the bag. Hence (B)	1

Section II
60 marks

Question 11 (15 marks)

11(a) (2 marks)
Outcomes Assessed: ME-F1.2/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Correctly identifies 2 and $\frac{11}{4}$ as important, or equivalent merit	1

Sample Answer:

$$\frac{3}{x-2} < 4$$

$$(x-2)^2 \times \frac{3}{(x-2)} < 4 \times (x-2)^2$$

$$3(x-2) < 4(x-2)^2$$

$$(x-2)(3-4x+8) < 0$$

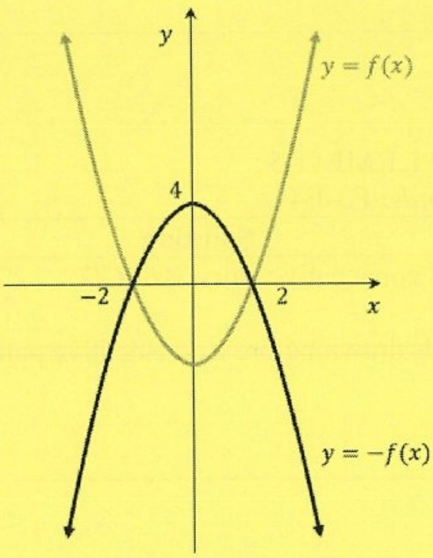
$$(x-2)(11-4x) < 0$$

Hence, $(-\infty, 2) \cup (\frac{11}{4}, \infty)$.

11 (b) (i) (1 mark)
Outcomes assessed: ME-F1.1/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct sketch	1

Sample Answer:

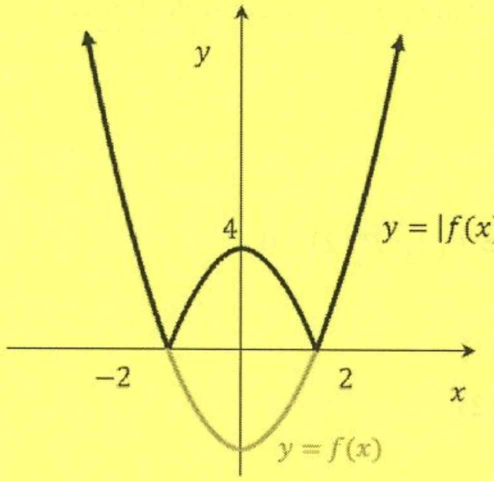


Disclaimer

11 (b) (ii) (1 mark)
Outcomes assessed: ME-F1.1/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct sketch	1

Sample Answer:



11(c) (2 marks)
Outcomes assessed: ME-A1.1/ME11-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Arranges the letters with the vowels not adjacent, but missing some arrangements OR • Find the number of arrangements where the vowels are adjacent	1

Sample Answer:

Method 1:
Arrange the 6 consonants in a line in 6! ways.
Arrange the vowels in the 7 positions between and beside the consonants in ${}^{7}P_{2}$ ways.
Total arrangements $= 6! \times {}^{7}P_{2} = 30\ 240$

Method 2:
Find the number of arrangements with the vowels together:
Arrange the vowels in a bubble in 2! ways
There are now 7 bubbles which can be arranged in 7! ways.
There are $2! \times 7!$ arrangements with the vowels together.
There are 8! arrangements without restrictions.
Total arrangements $= 8! - 2! \times 7! = 30\ 240$

11(d) (3 marks)
Outcomes Assessed: ME-F2.2/ME11-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Correctly find any two values of a, b and c, or equivalent merit	2
• Correctly find one value of a, b or c, or equivalent merit	1

Sample Answer:

$P(x) = ax^3 + bx^2 + c$

$P'(x) = 3ax^2 + 2bx$

Since double root at $x = 2 \Rightarrow P(2) = P'(2) = 0$

$P'(2) = 12a + 4b = 0$

$\Rightarrow \quad 3a + b = 0 \quad (1)$

$P(2) = 8a + 4b + c = 0 \quad (2)$

Since $P(-2) = -64$

$\Rightarrow \quad -8a + 4b + c = -64 \quad (3)$

$(2) - (3) \Rightarrow \quad 16a = 64$

$a = 4$

Sub $a = 4$ into (1)

$\Rightarrow \quad 12 + b = 0$

$b = -12$

Sub $a = 4$ and $b = -12$ into (2)

$\Rightarrow \quad 32 - 48 + c = 0$

$c = 16$

11 (e) (3 marks)
Outcomes Assessed: ME-T2/ME11-3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Obtains correct value for $\sin 2x$, or equivalent merit	2
• Attempts to arrive at a double angle, or equivalent merit	1

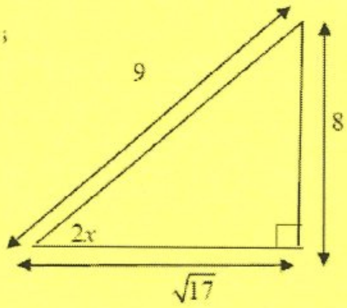
Sample Answer:

From $\cos x - \sin x = \frac{1}{3}$, squaring both sides

$$\cos^2 x - 2\cos x \sin x + \sin^2 x = \frac{1}{9}$$

$$\sin 2x = 1 - \frac{1}{9}$$

$$\sin 2x = \frac{8}{9}$$



$$\cot 2x = \frac{\sqrt{17}}{8}$$

11(f) (3 marks)
Outcomes Assessed: ME-C2/ME12-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Attempts to evaluate correct integral, or equivalent merit	2
• Attempts to obtain $\tan^{-1}\left(\frac{3x}{2}\right)$, or equivalent merit	1

Sample Answer:

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

$$= \frac{1}{9} \times \frac{3}{2}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}}$$

$$= \frac{1}{6}\left(\tan^{-1} 1 - \tan^{-1} 0\right)$$

$$= \frac{\pi}{24}$$

$$\therefore\ n = \frac{1}{24}$$

Question 12 (15 marks)

12 (a) (3 marks)
Outcomes Assessed: ME-C2/ME12-1
Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	3
• Provides a correct primitive in terms of u, or equivalent merit	2
• Attempts to use given substitution, or equivalent merit	1

Sample Answer:

$$u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$du = e^x dx$$

Also $u = 1 + e^x$ or $e^x = u - 1$

$$\int \frac{e^{3x}}{1+e^x}\, dx = \int \frac{e^{2x} \times e^x dx}{1+e^x}$$

$$= \int \frac{(u-1)^2 \times du}{1+u-1}$$

$$= \int \frac{u^2 - 2u + 1}{u}\, du$$

$$= \int \left(u - 2 + \frac{1}{u}\right) du$$

$$= \frac{u^2}{2} - 2u + \ln|u| + C$$

$$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln|1+e^x| + C$$

12 (b) (3 marks)
Outcomes Assessed: ME-C1.3/ME11-4
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Correctly finds $\frac{dy}{dt}$, or equivalent merit	2
• Correctly finds $\frac{dy}{dx}$, or equivalent merit	1

Sample Answer:

Using $x^2 + y^2 = 25$

$y = \sqrt{25 - x^2} \quad (y > 0)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$

Now $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = \frac{dy}{dx} . \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}} \times 1$

Now when $y = 4, \ x = 3$

$\frac{dy}{dt} = \frac{-3}{\sqrt{25 - 3^2}} \times 1$

$= -\frac{3}{4}$ (i.e. $\frac{3}{4}$ metres per second down the wall)

12 (c) (3 marks)
Outcomes assessed: ME-P1/ME12-1
Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	3
• Provides inductive step by assuming k (or equivalent) and using assumption to show true for $k+1$	2
• Verifies base case, $n=1$ or equivalent	1

Sample Answer:

$4^1+14=18$, 18 is divisible by 6, therefore true for $n=1$.

Assume true for $n=k$

ie $4^k+14=6M$, M is an integer

$4^k=6M-14$

Prove true for $n=k+1$

$4^{k+1}+14=6Q$, Q is an integer

$4^k.4+14$

$=(6M-14)4+14$

$=24M-56+14$

$=24M-42$

$=6(4M-7)$

$=6Q$. since $4M-7$ is an integer

By the principle of mathematical induction the statement is true for $n \geq 1$

12 (d) (i) (2 marks)
Outcomes Assessed: ME-S1.1/ME12-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains correctly either $\mathrm{E}(X)$ or $\mathrm{Var}(X)$	1

Sample Answer:

$\mathrm{E}(X)=np=100\times0.7=70$

$\mathrm{Var}(X)=np(1-p)=100\times0.7\times0.3=21$

12 (d) (ii) (2 marks)
Outcomes Assessed: ME-S1.1/ME12-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Evaluates an incorrect expression correctly, where the indices are in the incorrect order, or equivalent merit OR • Obtains correct expression but does not evaluate it	1

Sample Answer:

$$P(X=70)=\binom{100}{70}0.7^{70}\times 0.3^{30}$$
$$=0.086783...$$
$$=0.0868\ (3\text{ sf})$$

12 (d) (iii) (2 marks)
Outcomes Assessed: ME-S1.2/ME12-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Obtains correct z value OR • Obtains correct answer from incorrect z value	1

Sample Answer:

$\sigma=\sqrt{0.7\times 0.3}=\sqrt{0.21}$

$z=\dfrac{0.65-0.7}{\sqrt{0.21}}=-0.11$ (2 dp)

$P(z<-0.11)=0.4562$ (from z tables)

The probability that $\hat{p}<65\%$ is 0.46.

Also accept calculations using 0.645 (continuity correction):

$\sigma=\sqrt{0.7\times 0.3}=\sqrt{0.21}$

$z=\dfrac{0.645-0.7}{\sqrt{0.21}}=-0.12$ (2 dp)

$P(z<-0.12)=0.4522$ (from z tables)

The probability that $\hat{p}<65\%$ is 0.45.

Question 13 (15 marks)

13 (a) (2 marks)
Outcomes Assessed: ME-V1.2/ME12-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains correct magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ OR • Obtains angle using one or two incorrect magnitudes OR • Obtains an answer using sine instead of cosine with no further mistakes	1

Sample Answer:

$$|\overrightarrow{OA}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$|\overrightarrow{OB}| = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$\begin{aligned}\cos \angle AOB &= \frac{\binom{1}{-1} \cdot \binom{1}{4}}{\sqrt{2} \times \sqrt{17}} \\ &= \frac{1-4}{\sqrt{34}} \\ &= -\frac{3}{\sqrt{34}} \\ \angle AOB &= 120.963... \\ &= 121^\circ \text{ (nearest degree)}\end{aligned}$$

13 (b) (4 marks)
Outcomes Assessed: ME-C2/ME12-1 ME-C3.1/ME12-4
Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	4
• Attempts to evaluate correct integral, or equivalent	3
• Attempts to integrate the volume expression with correct primitive for $\cos^2 x$, or equivalent merit	2
• Obtains correct expression for the volume, or equivalent merit	1

Sample Answer:

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1-\cos x)^2 \, dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1-2\cos x+\cos^2 x\right) dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1-2\cos x+\frac{1+\cos 2x}{2}\right) dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{3}{2}-2\cos x+\frac{1}{2}\cos 2x\right) dx$$

$$= \pi\left[\frac{3x}{2}-2\sin x+\frac{1}{4}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \pi\left(\frac{3\pi}{4}-2-\left(\frac{3\pi}{8}-\sqrt{2}+\frac{1}{4}\right)\right)$$

$$= \pi\left(\frac{3\pi}{8}-\frac{9}{4}+\sqrt{2}\right) u^3.$$

13 (c) (4 marks)
Outcomes Assessed: ME-T3/ME12-3
Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	4
• Correctly writes $3\sin x - 4\cos x$ in the form $R\sin(x+\alpha)$ and finds one solution, or equivalent merit	3
• Finds R and α, or equivalent merit	2
• Finds the value of R or α, or equivalent merit	1

Sample Answer:

$$R = \sqrt{3^2 + 4^2} = 5, R > 0$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore 3\sin x - 4\cos x = 5\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right)$$

$$5\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right) = 2.5$$

$$\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right) = \frac{1}{2}$$

$$x - \tan^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{6}, \ \frac{5\pi}{6}$$

$$x = \frac{\pi}{6} + \tan^{-1}\left(\frac{4}{3}\right), \ \frac{5\pi}{6} + \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 1.451^c, 3.545^c \text{ (3 dp)}$$

$$= 1.451^c, -2.738^c \text{ since } -\pi \le x \le \pi$$

13 (d) (2 marks)
Outcomes Assessed: ME-C2/ME12-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains the correct primitive	1

Sample Answer:

$$f(x) = \int \frac{2}{4+x^2}\,dx$$

$$= 2 \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

Given $\left(2, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = \tan^{-1}(1) + C$

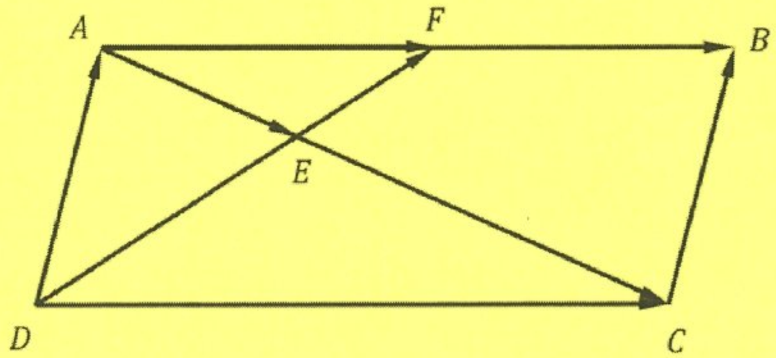
$$C = \frac{\pi}{4}$$

$$\therefore f(x) = \tan^{-1}\left(\frac{x}{2}\right) + \frac{\pi}{4}$$

13 (e) (3 marks)
Outcomes Assessed: ME-V1.2/ME12-2
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Express $\overrightarrow{AE}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$ and $\overrightarrow{AF}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$	2
• Express $\overrightarrow{AE}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$ or $\overrightarrow{AF}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$	1

Sample Answer:



Disclaimer

Let $\overrightarrow{DA} = \underset{\sim}{u}$ and $\overrightarrow{DC} = \underset{\sim}{v}$

In ΔADC, $\overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$ $\therefore$ $\overrightarrow{AC} = \overrightarrow{DC} - \overrightarrow{DA} = \underset{\sim}{v} - \underset{\sim}{u}$

Given $\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AC}$ $\therefore$ $\overrightarrow{AE} = \frac{2}{5}(\underset{\sim}{v} - \underset{\sim}{u})$

In ΔAED, $\overrightarrow{DA} + \overrightarrow{AE} = \overrightarrow{DE}$ $\therefore$ $\overrightarrow{DE} = \underset{\sim}{u} + \frac{2}{5}(\underset{\sim}{v} - \underset{\sim}{u}) = \frac{2}{5}\underset{\sim}{v} + \frac{3}{5}\underset{\sim}{u}$

Let $\overrightarrow{DF} = \alpha\overrightarrow{DE}$ and $\overrightarrow{AF} = \mu\overrightarrow{AB} = \mu\overrightarrow{DC} = \mu\underset{\sim}{v}$

In ΔDAF, $\overrightarrow{DA} + \overrightarrow{AF} = \overrightarrow{DF} = \alpha\overrightarrow{DE}$ $\therefore$ $\underset{\sim}{u} + \mu\underset{\sim}{v} = \alpha\left(\frac{2}{5}\underset{\sim}{v} + \frac{3}{5}\underset{\sim}{u}\right)$

Hence $\frac{3}{5}\alpha = 1$ and $\mu = \frac{2}{5}\alpha$ $\therefore$ $\alpha = \frac{5}{3}$ and $\mu = \frac{2}{3}$

$\therefore$ $\overrightarrow{AF} = \frac{2}{3}\overrightarrow{DC}.$

Question 14 (15 marks)

14 (a) (2 marks)
Outcomes Assessed: ME-A1.2/ME11-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Obtains correct substitution for $x = 1$ or $x = -1$	1

Sample Answer:

Using the expansion

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$$

When $x = 1$

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} \qquad (1)$$

When $x = -1$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \ldots - \binom{n}{n} \qquad (2)$$

(1) – (2):

$$2^n = 2\left(\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots + \binom{n}{n}\right)$$

$$\therefore\ 2^{n-1} = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots + \binom{n}{n}$$

14 (b) (3 marks)
Outcomes Assessed: ME-A1.2/ME11-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Obtains $C_1 = 5\times\binom{15}{5}2^{10}3^5$ or $C_2 = 4\times\binom{15}{4}2^{11}3^4$	2
• Obtains the general term for the expression $\left(2x+\frac{3}{x^2}\right)^{15}$ and attempts to find the independent terms in the expansion, or equivalent merit	1

Sample Answer:

General term for $\left(2x+\frac{3}{x^2}\right)^{15} \Rightarrow T_{k+1} = \binom{15}{k}(2x)^{15-k}\left(\frac{3}{x^2}\right)^k$

$$= \binom{15}{k}2^{15-k}3^k x^{15-3k}$$

There are two terms which are independent of x in the expansion of $\left(5+\frac{4}{x^3}\right)\left(2x+\frac{3}{x^2}\right)^{15}$.

$5\times x^0$ when $k=5$ $(15-3k=0)$ $\Rightarrow C_1 = 5\times\binom{15}{5}2^{10}3^5$

and

$\frac{4}{x^3}\times x^3$ when $k=4$ $(15-3k=3)$ $\Rightarrow C_2 = 4\times\binom{15}{4}2^{11}3^4$

Therefore the term independent of x in the expansion of $\left(5+\frac{4}{x^3}\right)\left(2x+\frac{3}{x^2}\right)^{15}$

is $C_1 + C_2 = 5\times\binom{15}{5}2^{10}3^5 + 4\times\binom{15}{4}2^{11}3^4 = 4\ 641\ 960\ 960$

14 (c) (i) (3 marks)
Outcomes Assessed: ME-V1.3/ME12-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains expression for $\underset{\sim}{v}(t)$ with constants OR • Obtains expressions for $\dot{x}$ and $\dot{y}$ with constants	1

Sample Answer:

$$\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$\underset{\sim}{v}(t) = \begin{pmatrix} c_1 \\ -10t + c_2 \end{pmatrix}$$

Let $t = 0$, $\underset{\sim}{v}(0) = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$

$$\therefore \begin{pmatrix} c_1 \\ -10(0) + c_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$$

$$\therefore c_1 = 20,\ c_2 = 20\sqrt{3}$$

$$\therefore \underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 20\sqrt{3} - 10t \end{pmatrix}$$

14 (c) (ii) (2 marks)
Outcomes Assessed: ME-V1.3/ME12-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains the time to the top of the flight but doesn't double the answer, or equivalent merit	1

Sample Answer:

At the top of the flight $\underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 0 \end{pmatrix}$, so

$$20\sqrt{3} - 10t = 0$$
$$t = 2\sqrt{3}$$

$\therefore$ time of flight $= 2(2\sqrt{3}) = 4\sqrt{3}$ seconds.

14 (c) (iii) (2 marks)
Outcomes Assessed: ME-V1.3/ME12-2
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Finds either the maximum or minimum magnitude	1

Sample Answer:

The maximum magnitude of velocity occurs at the point of projection and impact, while the minimum occurs at the top of the flight.

$$\left|\underset{\sim}{v}\right|_{\max} = \sqrt{20^2 + \left(20\sqrt{3}\right)^2} = 40 \text{ m/s}$$

$\left|\underset{\sim}{v}\right|_{\min} = 20$ m/s (the horizontal velocity is constant and the vertical velocity is zero)

$$\therefore \left|\underset{\sim}{v}\right|_{\max} = 2\left|\underset{\sim}{v}\right|_{\min}$$

14 (d) (i) (1 mark)
Outcomes Assessed: ME-C3.2/ME12-4
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	1

Sample Answer:

$$\begin{aligned} LHS &= \frac{\tan^3 t}{\cos^4 t} \\ &= \sec^2 t \sec^2 t \tan^3 t \\ &= \sec^2 t\left(1+\tan^2 t\right)\tan^3 t \\ &= \sec^2 t\left(\tan^3 t + \tan^5 t\right) \\ &= RHS \end{aligned}$$

14(d) (ii) (3 marks)
Outcomes Assessed: ME-C3.2/ME12-4
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Obtains correct expression of y^2 as a function of x	2
• Separates the variable and integrate for x or y	1

Sample Answer:

$$\frac{dA}{dt} = \frac{\tan^3 t}{A\cos^4 t}$$

$$\int_1^A A\ dA = \int_{\frac{\pi}{4}}^t \frac{\tan^3 t}{\cos^4 t}\,dt$$

$$\frac{1}{2}\left[A^2\frac{2}{2}\right]_1^A = \int_{\frac{\pi}{4}}^t \sec^2 t\,(\tan^5 t + \tan^3 t)\,dt$$

$$\frac{1}{2}(A^2 - 1) = \left[\frac{\tan^6 t}{6} + \frac{\tan^4 t}{4}\right]_{\frac{\pi}{4}}^t$$

$$A^2 - 1 = 2\left(\left(\frac{\tan^6 t}{6} + \frac{\tan^4 t}{4}\right) - \left(\frac{1}{6} + \frac{1}{4}\right)\right)$$

$$A^2 = \frac{\tan^6 t}{3} + \frac{\tan^4 t}{2} + \frac{1}{6}$$

$$A = \sqrt{\frac{2\tan^6 t + 3\tan^4 t + 1}{6}} \quad \text{since } A \geq 0$$

BLANK PAGE

BLANK PAGE