2021
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

CSSA

DO NOT REMOVE PAPER FROM EXAM ROOM

Centre Number

Student Number

Mathematics Advanced

Morning Session
Thursday, 29 July 2021

General Instructions

- Reading time – 10 mins
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- Use Multiple Choice Answer Sheet provided
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 100

Section I Pages 2-6
10 marks
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II Pages 7-33
90 marks
- Attempt Questions 11 - 35
- Allow about 2 hours and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1 - 10.

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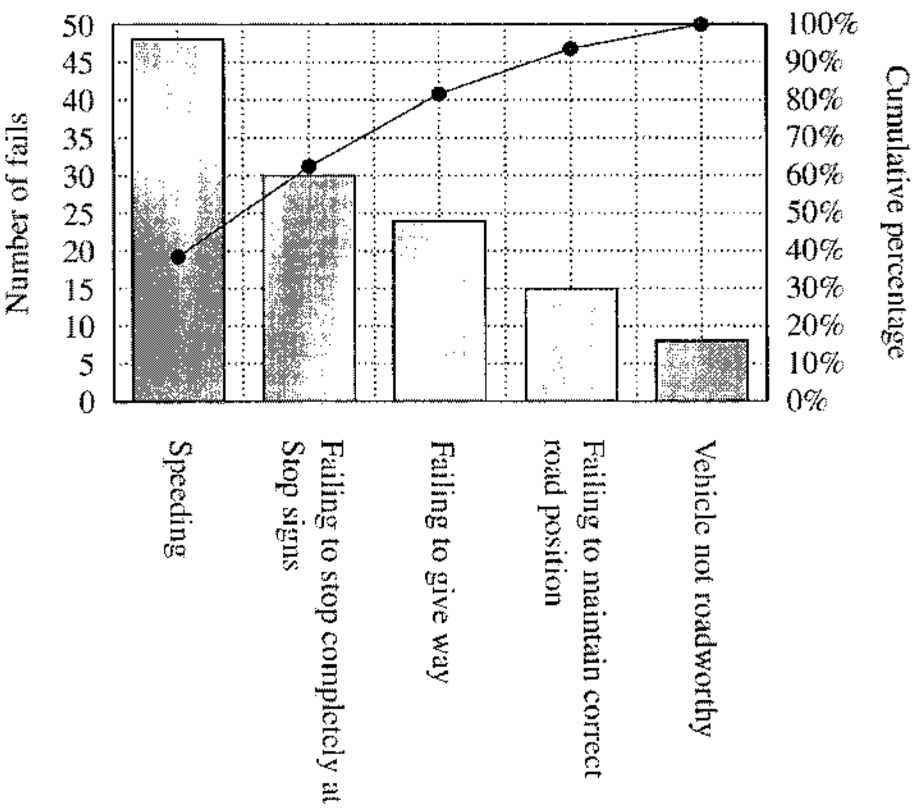
1 The Pareto chart below shows the reasons why learner drivers at a particular testing centre failed their driving test.



Approximately what percentage of learner drivers failed because they did not stop completely at stop signs?

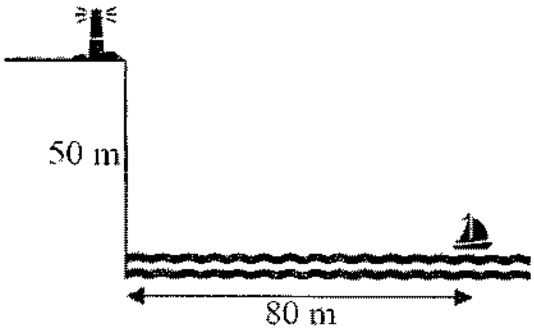
A. 75%

B. 62%

C. 40%

D. 24%

2 A lighthouse sits on top of a 50 metre high cliff. A yacht is 80 metres out to sea below the lighthouse.



NOT TO SCALE

What is the angle of depression of the yacht from the base of the lighthouse to the nearest degree?

A. 32°

B. 39°

C. 51°

D. 58°

3 The results of a test are normally distributed with a mean of 67 and a standard deviation of 7. Two students sit the test late and score 60 and 72 respectively.

Which row of the table below describes the impact of these new scores on the mean and standard deviation of the test?

	Mean	*Standard Deviation*
A.	Decreases	Increases
B.	Decreases	Decreases
C.	Increases	Increases
D.	Increases	Decreases

4 The table below shows the probability distribution of a discrete random variable X which has mean -0.45.

x	-2	-1	0	1	2
$P(X=x)$	0.1	a	0.2	0.15	b

What are the values of a and b?

A. $a=0.2,\ b=0.35$

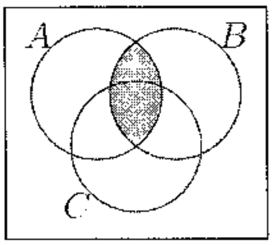
B. $a=0.3,\ b=0.25$

C. $a=0.4,\ b=0.15$

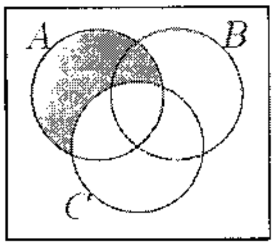
D. $a=0.5,\ b=0.05$

5 In which of the Venn diagrams below is the set $A \cap \left(B \cup \overline{C}\right)$ shaded, where $\overline{C}$ is the complement of C?

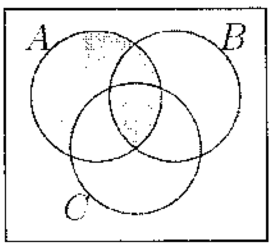
A.



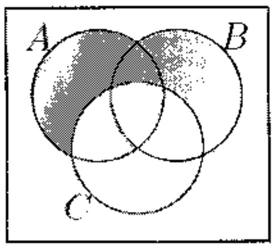
C.



B.



D.



6 Penny and Lara decided to buy a country property. After one year, Penny and Lara noticed that the noxious weed, Paterson's curse, had started growing and they counted 600 individual weeds.

By the end of the second year they counted 1300 weeds, and by the end of the third year the property contained 2000 weeds.

Assuming the weeds continue to grow at this rate, how many weeds could be expected on the property after 10 years?

A. 6000 weeds

B. 6900 weeds

C. 7000 weeds

D. 37500 weeds

7 What is the best way to describe the relation $y=(x+1)^3$?

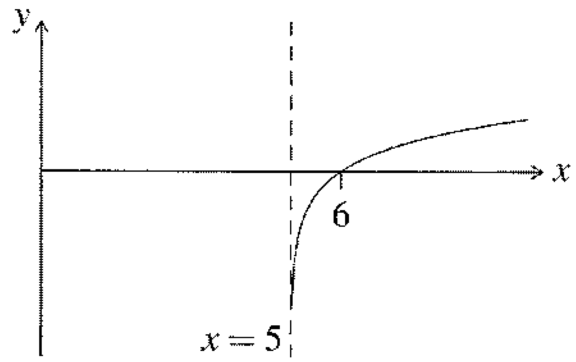
A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

8 Which equation best represents the following graph?



A. $y=\log_5(x-6)$

B. $y=\log_3(x-6)$

C. $y=\log_3(x-5)$

D. $y=\log_6(x-3)$

9 Which of the following will give the correct equation of the tangent to the curve $y = f(x)$ at the point $x = a$?

A. $y - f(a) = \left(\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \right)(x - a)$

B. $y - f(a) = \left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right)(x - a)$

C. $y - f(x) = \left(\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \right)(x - a)$

D. $y - f(x) = \left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right)(x - a)$

10 How many points of intersection are there between the curves $y = 2^x$ and $y = x^2$?

A. 0

B. 1

C. 2

D. 3

Section II

90 marks
Attempt Questions 11 - 35
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 34–35. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (3 marks)

Emma is saving for a new car. She deposits \$2200 into an annuity account at the end of every year, for four years. The account pays 3% per annum interest, compounding annually. **3**

The table shows future values of an annuity of \$1.

Future values of an annuity of \$1

Years	Interest rate per annum						
	1%	**1.5%**	**2%**	**2.5%**	**3%**	**3.5%**	**4%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0150	2.0200	2.0250	2.0300	2.0350	2.0400
3	3.0301	3.0452	3.0604	3.0756	3.0909	3.1062	3.1216
4	4.0604	4.0909	4.1216	4.1525	4.1836	4.2149	4.2465
5	5.1010	5.1523	5.2040	5.2563	5.3091	5.3625	5.5256

How much interest does Emma earn on her investment over the four years?

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Question 12 (2 marks)

Solve $\dfrac{729}{4\sqrt{3}-\sqrt{27}}=3^x$. **2**

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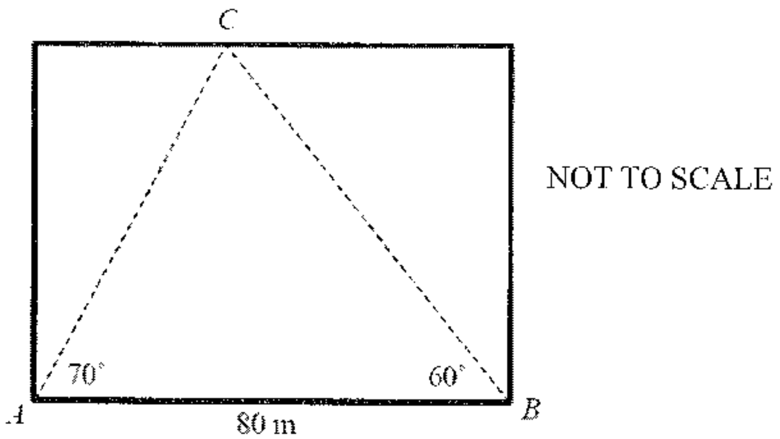
Question 13 (4 marks)

Ann, Bo Lai and Corinne are positioned at points A, B and C respectively on a rectangular playground as shown in the diagram below. The length of the playground is 80 metres, $\angle ABC = 60°$ and $\angle BAC = 70°$.



(a) Calculate the distance from Bo Lai to Corinne. Give your answer correct to the nearest metre. **2**

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(b) Calculate the width of the playground. Give your answer correct to the nearest metre. **2**

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Question 14 (3 marks)

Consider the function $f(x) = ax + b$ where a and b are constants. The function satisfies $f(-3) = 9$ and $f(5) = -7$.

(a) Find the values of a and b. **2**

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(b) Solve the equation $f\left(f(x)\right) = 0$. **1**

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Question 15 (3 marks)

Steve and Stella take out a loan of $50 000 for home renovations at 3% per annum compounded monthly. They make regular repayments of $750 at the end of each month. **3**

The recurrence relation to model this situation is given by the formula

$$A_n = A_{n-1}(1.0025) - 750.$$

How much have Steve and Stella paid off the amount they borrowed, after the first three repayments?

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Question 16 (2 marks)

If $y = \log_e x$, show that $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0.$ **2**

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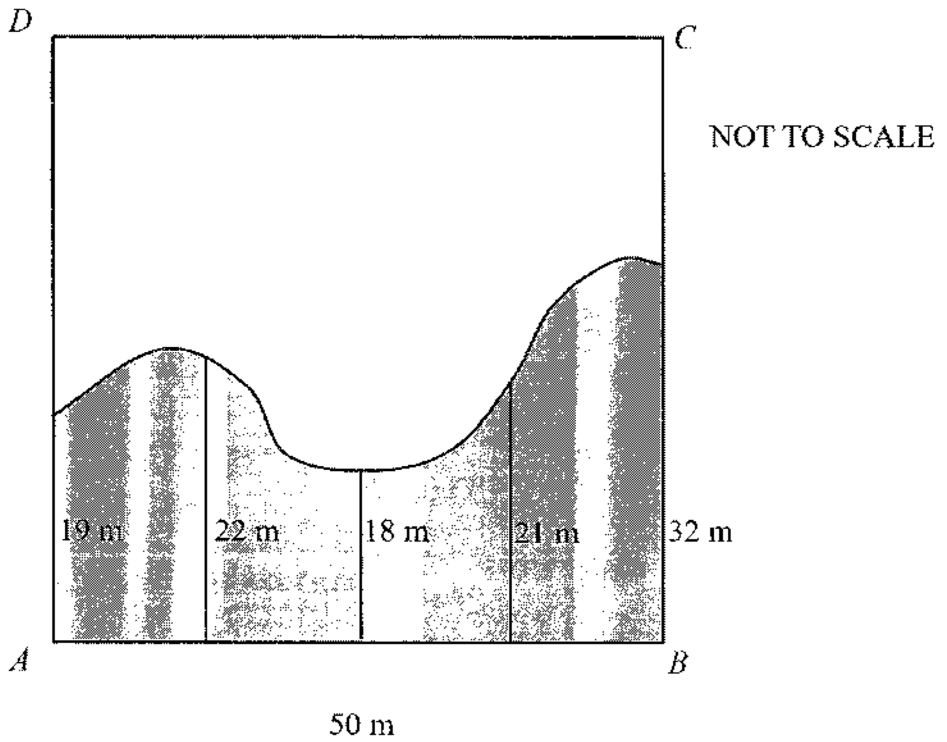
Question 17 (3 marks)

Archaeologists are excavating a square plot *ABCD* measuring 50 metres by 50 metres.

The plot is divided into four sections of equal width as shown in the diagram below.

At the end of the first phase of digging, the shaded area has been excavated. Measurements made to the edges of the excavated area are also shown on the diagram.



(a) Use the trapezoidal rule with each of the measurements given to estimate the shaded area. **2**

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(b) Express the shaded area as a percentage of the total area. Give your answer correct to two decimal places. **1**

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Question 18 (4 marks)

A football coach is studying the relationship between the weight in kilograms of players and their vertical jump in metres. The coach believes that players who weigh less are able to jump higher.

The data from ten players are shown in the table below.

weight w	84.5	83.3	58.7	74.1	70.7	72.3	75.8	71.8	63.2	65.9
vertical jump v	21.9	20.5	20.8	19.9	20.6	18.8	22.8	20.6	20.0	21.8

(a) Find the equation of the least-squares regression line. Give each coefficient correct to two decimal places. **2**

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(b) By calculating Pearson's correlation coefficient of this data, correct to three decimal places, justify whether the data confirms the association between weight and vertical jump. **2**

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Question 19 (5 marks)

Consider $f(x)=\begin{cases} x^2-2x+2 & \text{for } -3<x\le 1, \\ x+1 & \text{for } 1<x<6. \end{cases}$

(a) Determine whether $f(x)$ is continuous at $x=1$. **2**

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(b) Evaluate $f(-1)+f(2)$. **1**

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(c) Find the range of $f(x)$. **2**

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Question 20 (3 marks)

A scientist is modelling a locust plague that is swarming in Uganda. The area affected by locusts is given by $A = 800 \times (1.25)^n$ hectares, where n is the number of weeks after the initial observation.

(a) Find the affected area after four weeks. Give your answer to the nearest hectare. **1**

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(b) Find the smallest number of weeks until the locust plague overtakes Uganda, given Uganda is approximately 24.1 million hectares. **2**

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Question 21 (4 marks)

A fountain is designed so that the water projected from it forms a parabolic arc over a walkway. **4**

If x metres represents the horizontal distance in metres from the base of the fountain and y metres the vertical height of the water above the ground, the path of the water can be modelled by the equation $y = x(6 - x)$, where $0 \le x \le 6$.

The average person is 1.71 m tall. If safety engineers stipulate that the average person needs 1.3 m width to walk comfortably, determine the maximum number of average-sized people that can comfortably walk under the fountain, side-by-side, without getting wet.

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Question 22 (4 marks)

(a) Show that $\frac{d}{dx}(x\cos x - \sin x) = -x\sin x.$ **2**

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(b) Hence, or otherwise, evaluate **2**

$$\int_0^{\frac{\pi}{4}} x\sin x\, dx.$$

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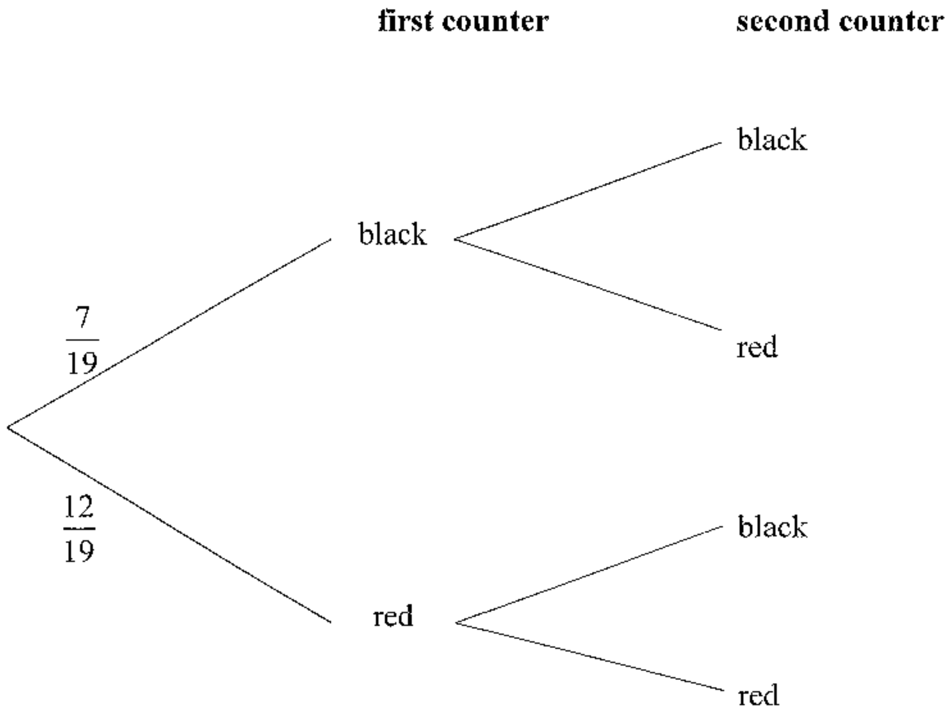
Question 23 (3 marks)

A bag contains 7 black and 12 red counters.

Lucas selects two counters from the bag at random without replacement.

(a) Complete the probability tree below, showing the probability of each of the four possible selections for the second counter. **1**



(b) By using the probability tree, or otherwise, determine whether Lucas is more likely to select two counters of the same colour or two counters of different colours. **2**

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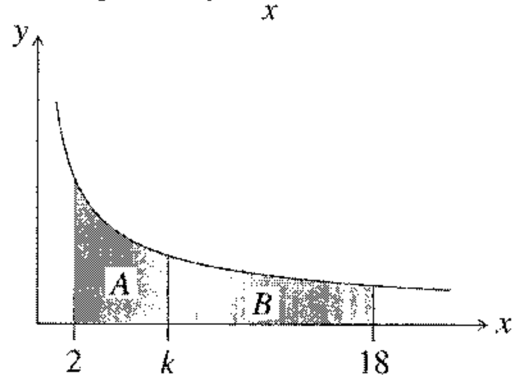
Question 24 (3 marks)

The diagram below shows the hyperbola $y = \frac{1}{x}$. **3**



Find k such that area A and area B are equal.

Question 25 (4 marks)

A small rural school has counted the number of Year 12 students in Art and Biology. They found that: **4**

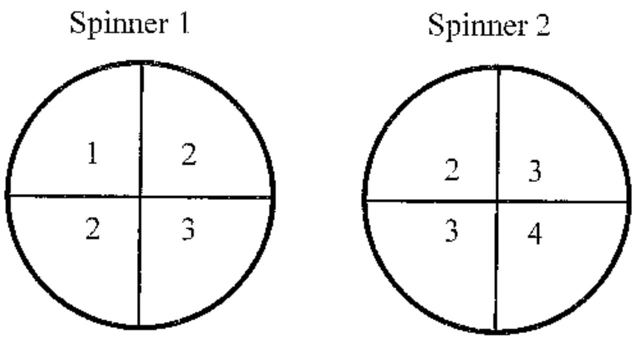
- 5 students study Art only,
- 12 students study Biology only,
- 3 students study neither,
- k students study both.

Let A be the event that a Year 12 student studies Art, and let B be the event that a Year 12 student studies Biology.

Determine the value of k such that the events A and B are independent.

Question 26 (4 marks)

A spinner is labelled $\{1, 2, 2, 3\}$ and another spinner is labelled $\{2, 3, 3, 4\}$, as shown below.



Let X denote the random variable for the sum of the two numbers when the spinners are spun.

(a) Complete the probability distribution table of X. **2**

x	3	4	5	6	7
$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$		

(b) Find $P\left(X = 5 \mid X \geq 4\right)$. **2**

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Question 27 (4 marks)

After taking a break at a rest stop, Sergio starts driving down a highway. His car's velocity after t seconds is given by the equation $v(t) = 0.3t^2$ m/s.

(a) Find how long it takes the car's velocity to reach 30 m/s. **1**

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(b) Find how far Sergio's car has travelled during this time, in metres. **2**

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(c) Find the average speed of the car, in m/s, from when Sergio starts driving down the highway from the rest stop to when it reaches 30 m/s. **1**

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Question 28 (4 marks)

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 0, \\ \frac{1}{3}x^2\left(4 - x^2\right) & \text{for } 0 \leq x \leq 1, \\ 1 & \text{for } x > 1. \end{cases}$$

(a) Find $P(X > 0.6)$. **2**

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(b) Find the probability density function $f(x)$ of X. **1**

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(c) Determine the mode of the distribution. Give your answer correct to three decimal places. **1**

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Question 29 (4 marks)

Sketch the graph of the curve $y = x^3 + 3x^2 - 9x$, labelling the stationary points and the point of inflection. Do NOT determine the x-intercepts of the curve. **4**

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Question 29 continues on the next page

Question 29 (continued)

End of Question 29

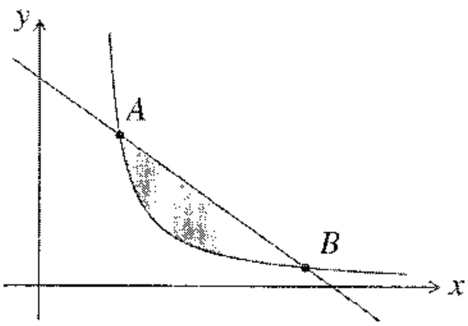
Question 30 (2 marks)

Four times the sum of the first two terms of an infinite geometric progression is equal to the limiting sum. **2**

Determine the possible values of the common ratio.

Question 31 (4 marks)

The diagram below shows a part of the graphs of $y = 5 - x$ and $y = \dfrac{2}{x-2}$.



(a) Show that the x-coordinates of the points of intersection A and B are 3 and 4 respectively. **1**

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(b) Find the exact area of the shaded region. **3**

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Question 32 (7 marks)

The town of Wilcannia, in Western NSW, and Kempsey, near the coast of NSW, have roughly the same latitude. The historical averages of their daily maximum temperatures are listed in the table below.

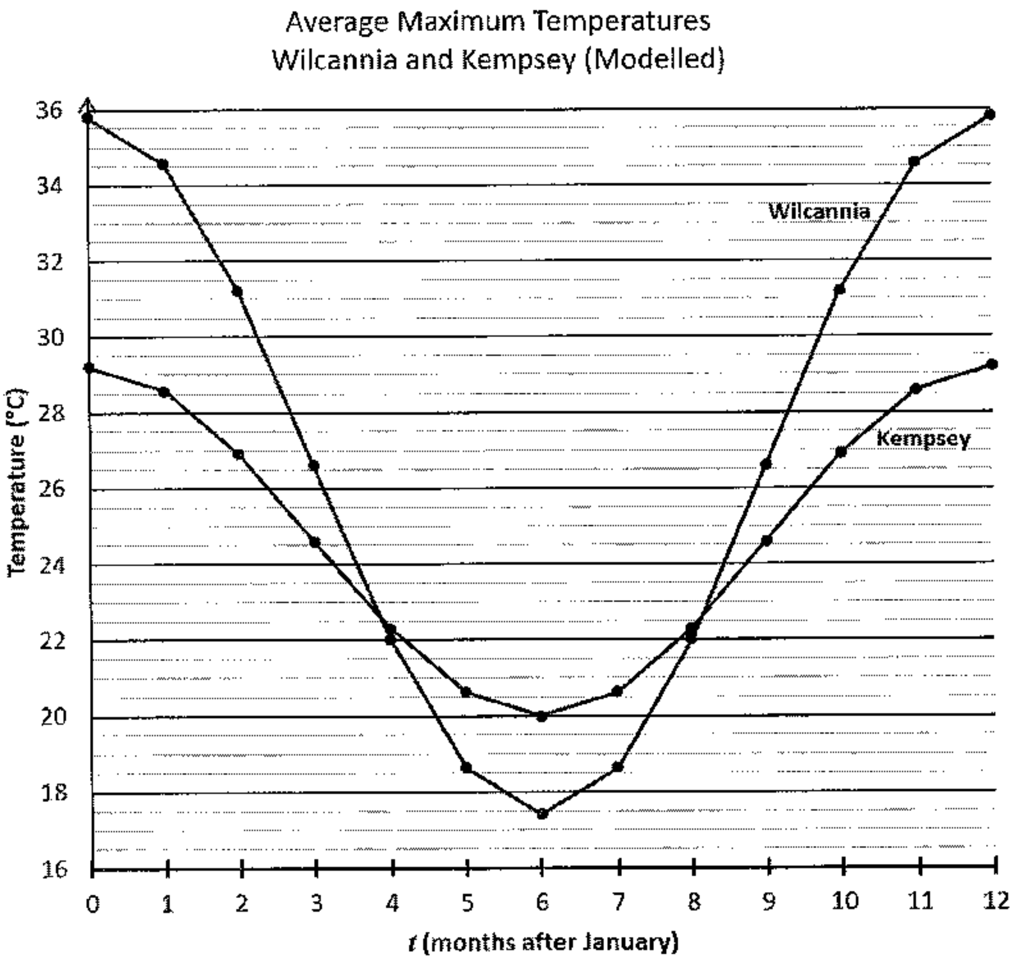
Historical averages of daily maximum temperatures in Wilcannia and Kempsey

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Wilcannia	35.5	34.6	31.3	26.2	21.3	17.7	17.1	19.5	23.5	27.5	31.2	34.0
Kempsey	29.2	28.8	27.8	25.5	22.6	20.0	19.7	21.3	24.0	25.6	27.1	28.5

(Source: http//www.bom.gov.au/climate/averages)

A model of the daily maximum temperature of Kempsey t months after January is given by

the function $K(t) = 24.6 + 4.6\cos\left(\frac{\pi t}{6}\right)$.



Question 32 continues on the next page

Question 32 (continued)

(a) There are small differences between the historical averages of the daily maximum temperatures and the values generated by the cosine function model. What is the difference between the recorded historical average for Kempsey in September and the temperature predicted by the model? **2**

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(b) Express $W(t)$ as a function of t without reference to $K(t)$. **2**

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(c) With reference to the variables in the two model equations, compare the average daily maximum temperatures for the two towns. **3**

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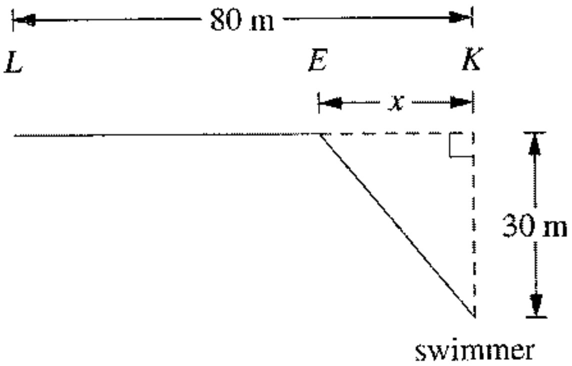
Question 33 (5 marks)

Kevin is a lifeguard on duty when he notices a swimmer in need of rescue. The swimmer is 30 m out from shore in a direct line from the kiosk at K. Kevin is 80 m down the beach at the lifeguard lookout L as indicated in the diagram below. **5**



Kevin runs along the edge of the water from L to E then swims to the swimmer. Kevin can run at speed of 8 m/s and swim at a speed of 2 m/s. Let the distance from the kiosk K to the point of entry E into the water be x metres.

How far along the shore should Kevin run before entering into the water in order to reach the swimmer in the fastest possible time? Give your answer to the nearest metre.

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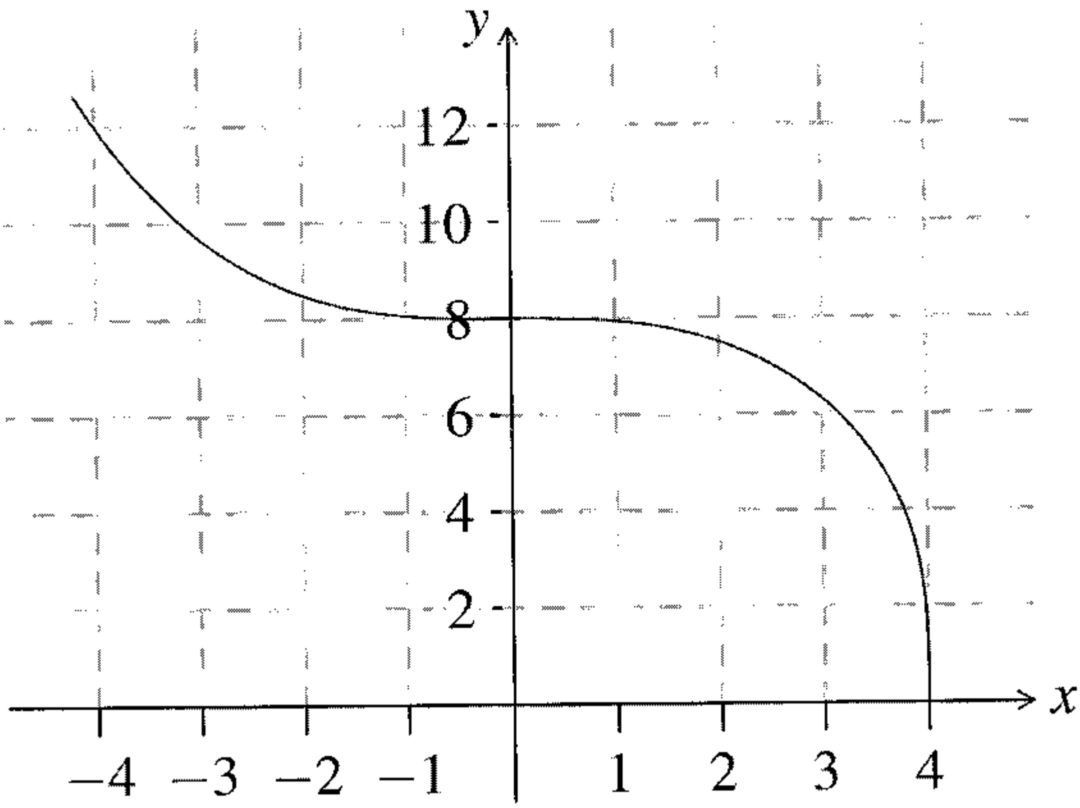
Question 33 continues on the next page

Question 33 (continued)

End of Question 33

Question 34 (3 marks) **3**

The diagram below shows the graph of $y = g(x)$.



Sketch the curve of $y = 10 - g(4x)$ on the same set of axes.

Question 35 (3 marks)

Determine the values of a such that the curve $y = x^3 + ax^2 - 1$ has three distinct x-intercepts. **3**

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End of examination

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Section II extra writing space

If you use this space, clearly indicate which question you are answering by writing the question number before beginning the response.

EXAMINERS

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