Neap

HSC Trial Examination 2020

Mathematics Advanced

General Instructions	
	• Reading time – 10 minutes • Working time – 3 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 100	Section I – 10 marks (pages 2–4) • Attempt Questions 1–10 • Allow about 15 minutes for this section Section II – 90 marks (pages 5–29) • Attempt Questions 11–28 • Allow about 2 hours and 45 minutes for this section

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2020 HSC Mathematics Advanced Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. What is the value of $\operatorname{cosec}\frac{\pi}{3}$ correct to three significant figures?

(A) 1.00

(B) 1.15

(C) 1.41

(D) 2.00

2. What is the amplitude of $f(x) = -3\sin\left(2x + \frac{\pi}{3}\right)$?

(A) $-\pi$

(B) -3

(C) 3

(D) π

3. What is the natural domain of $f(x) = \frac{1}{e^x}$?

(A) $(-\infty, \infty)$

(B) $[0, \infty)$

(C) $(0, \infty)$

(D) $(-\infty, 0]$

4. What is the equation of the tangent to $y = x^2 - 3$ at $x = -1$?

(A) $y = -2x - 4$

(B) $y = 2x - 4$

(C) $y = \frac{x}{2} - \frac{3}{2}$

(D) $y = -\frac{x}{2} - \frac{3}{2}$

5. Which one of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

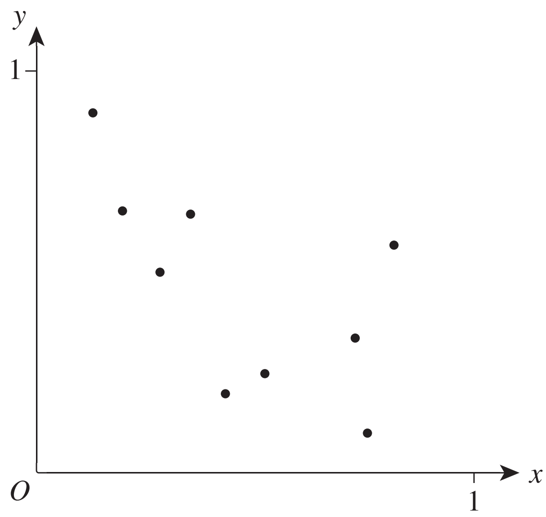
(A) $\left[\frac{1}{2}, 2\right]$

(B) $\left(\frac{1}{2}, 2\right)$

(C) $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

(D) $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

6. The scatter plot relates the quantities x and y.



Which one of the following values is the most appropriate Pearson's correlation coefficient for x and y?

(A) -0.97

(B) -0.63

(C) 0.12

(D) 0.55

7. The graph of $y = f(x)$ has a stationary point at $(2, -3)$.

Which one of the following is a guaranteed stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

(A) $(1, -2)$

(B) $(1, 2)$

(C) $(4, -2)$

(D) $(4, 2)$

8. Which one of the following equations is NOT correct?

(A) $\int x(x^2-1)^2\,dx = \frac{(x^2-1)^3}{6} + c$

(B) $\int_{-3}^{3} \sqrt{9-x^2}\,dx = \frac{9\pi}{2}$

(C) $\int_{-1}^{1} 3^x\,dx = \frac{1}{\ln 3}\left(3 - \frac{1}{3}\right)$

(D) $\int_{-5}^{5} 4x^4 - x^3 + \cos x\,dx = 0$

9. A particle moves according to the equation $x = t^2 - \ln(t+1)$, where $t \geq 0$.

How may times does the particle come to rest?

(A) 0
(B) 1
(C) 2
(D) 3

10. An endurance test requires participants to consistently jog around a 400 m track. A participant passes the test if they successfully jog 1 full lap. A participant is said to be 'Very Fit' if they successfully jog 3 full laps.

Where X is the number of full laps a participant successfully jogs, the distribution function of X is $P(X = x) = p(x) = 0.2(0.8)^k$ for $k = 0, 1, 2, \ldots$

What is the probability that a participant who has passed the test is Very Fit; that is, what is $P(X \geq 3 | X \geq 1)$?

(A) 0.288
(B) 0.512
(C) 0.64
(D) 0.8

Section II

90 marks
Attempt Questions 11–28
Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

What angle does the line $-x + 4y + 12 = 0$ make with the positive x-axis? Round to the nearest minute. **2**

Question 12 (3 marks)

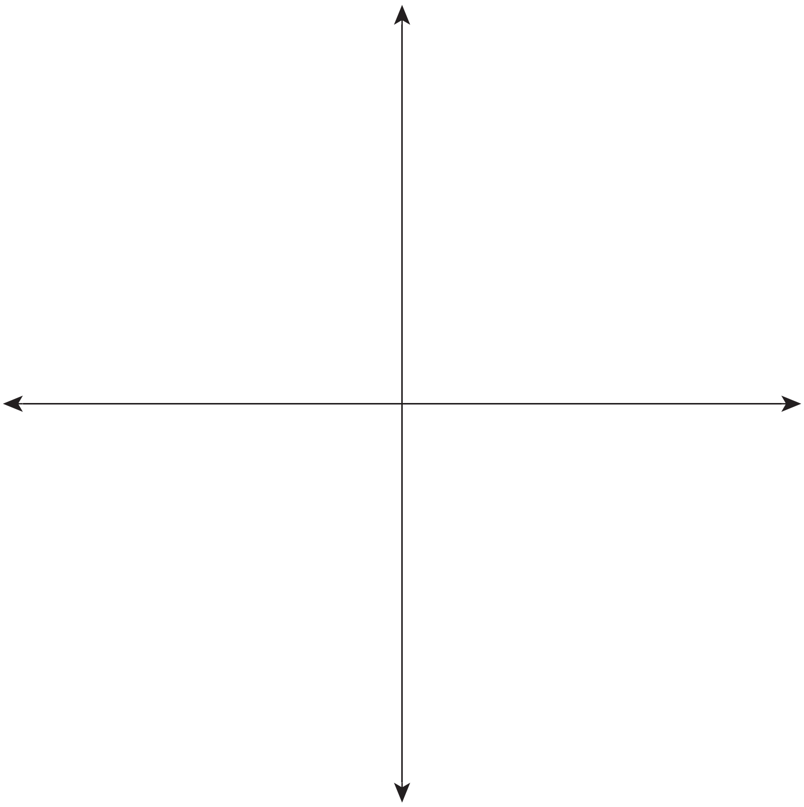
Differentiate the following expressions.

(a) $\log_2(\cos x)$ **1**

(b) $3^x e^x$ **2**

Question 13 (3 marks)

Sketch $y = \frac{3}{x+2} + 2$ on the axes below, showing all intercepts with the coordinate axes and all asymptotes. Show your working. **3**



Question 14 (3 marks)

Eye colour is recorded in a population of 266 male and female subjects.

(a) Complete the two-way table for the data collected. **2**

	Male	*Female*	*Total*
Brown	98		190
Other		37	76
Total	137		

(b) A random person is chosen from this population. **1**

Given that this person's eye colour is brown, what is the probability that they are female?

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Question 15 (2 marks)

An amount of money is invested in an account that earns interest at a rate of r per annum, compounding monthly. The corresponding effective annual rate of interest to this is 11.27%. **2**

Calculate the value of r as a percentage correct to two decimal places.

Question 16 (3 marks)

Find the value of k such that $\int_{-5}^{-2} \frac{x^2}{x^3 - 2} dx = \ln k$. **3**

Question 17 (16 marks)

Let $f(x) = (x+2)(x-2)^3$.

(a) Write down the x-intercepts of $y = f(x)$. **1**

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(b) Show that $f'(x) = 4(x-2)^2(x+1)$ and $f''(x) = 12x(x-2)$. **2**

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(c) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. Justify your answers fully. **3**

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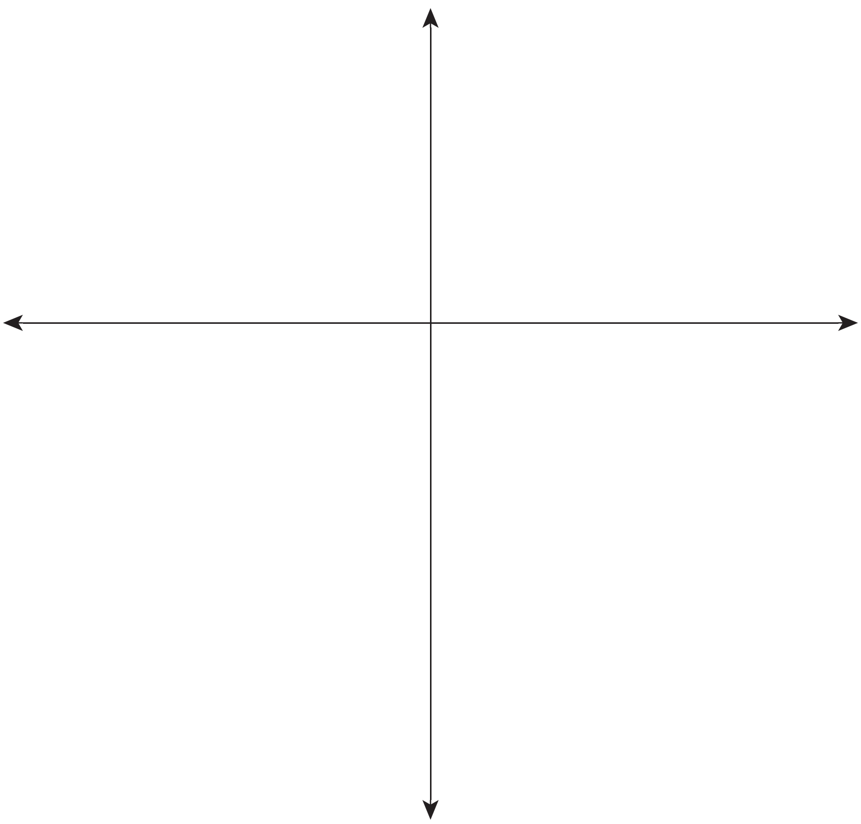
Question 17 continues on page 11

Question 17 (continued)

(d) Find the coordinates of all points of inflection of $y = f(x)$. **2**

(e) Sketch the graph of $y = f(x)$ on the axes below, showing all FOUR of the points found in parts (a), (b), and (c). **2**



Question 17 continues on page 12

Question 17 (continued)

(f) Does $y = f(x)$ have a global maximum or global minimum on its natural domain? If so, specify where. **1**

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(g) State, in the correct order, the transformations required to obtain the graph of $y = f\left(2\left(x - \frac{1}{4}\right)\right)$. **2**

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(h) On the set of axes provided in part (e), sketch the graph of $y = f\left(2\left(x - \frac{1}{4}\right)\right)$, showing its x-intercepts, stationary points and inflection points. **3**

End of Question 17

Question 18 (4 marks)

Consider the following extract from a table of FUTURE value interest factors, generated through the formula $A = P(1 + r)^n$.

n	*4%*	*8%*
0	1.0000	1.0000
1	1.0400	1.0800
2	1.0816	1.1664
3	1.1249	1.2587
4	1.1699	1.3605

(a) In an ordinary annuity, deposits are made at the end of every period. **2**

Calculate the PRESENT value of a 2 year ordinary annuity at rate 8% per annum, compounding every half year. Round your answer to the nearest cent.

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Question 18 continues on page 14

Question 18 (continued)

(b) A savings account is opened with deposits made at the end of each year. The interest rate is 8% per annum with annual compounding interest. Immediately after the fifth deposit, the amount in the account is $1000.00. **2**

What is the amount of the annual contribution that must be made to achieve this? Round your answer to the nearest cent.

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End of Question 18

Question 19 (4 marks)

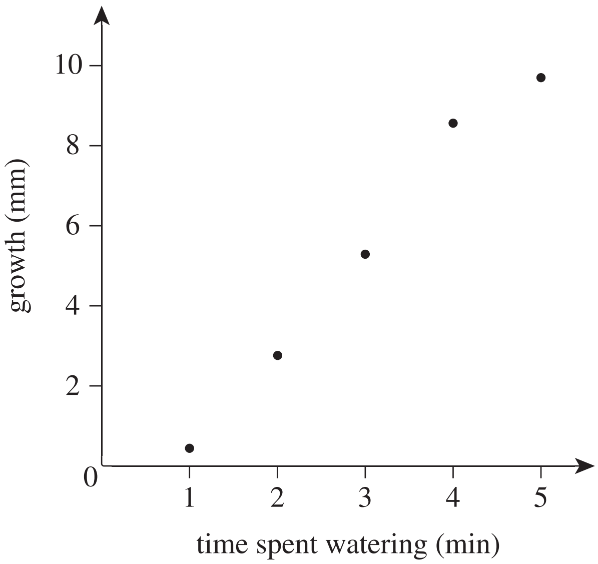
(a) Find all solutions of $\tan\left(2x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$ for all x in the interval $[0, 2\pi]$. **3**

(b) The graph of $y = \tan\left(2x - \frac{\pi}{3}\right)$ is plotted along with its vertical asymptotes. **1**

Identify any ONE of the vertical asymptotes.

Question 20 (3 marks)

A study compared the growth of a plant in one day with the time spent watering the plant on that day. The measurements were taken on five separate days and are recorded in the scatter plot below.



The least squares regression line for this data is $y = -0.1263 + 2.0694x$. Do NOT prove this.

(a) A student attempts to use this model to determine how much their plant will grow in a day if they water it for 10 minutes on that day. **1**

How much growth does the model predict?

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(b) Is the prediction made in part (a) convincing? Justify your answer. **2**

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Question 21 (3 marks)

In an arithmetic sequence, the fifth term is 9 and the twenty-first term is 425. **3**

What is the smallest value of n such that the sum of the first n terms in the sequence is at least 10 487?

Question 22 (6 marks)

Alex opened up a new investment fund that gains interest at 6% per annum, compounding quarterly. He makes regular deposits of \$4000 at the end of every three months. The first deposit was made immediately after the account was opened.

Let A_n be the amount of money in Alex's fund after n quarter years, for $n \geq 0$.

(a) Show that if Alex maintains his deposits for nine full years, his fund will have reached a value of \$195 940.44 to the nearest cent. **3**

(b) What is the single sum required when invested under the same conditions that would reach this value after 9 years? Give your answer to the nearest cent. **1**

Question 22 continues on page 19

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Question 22 (continued)

(c) After how many full months will Alex's fund reach \$500 000? **2**

End of Question 22

Question 23 (10 marks)

The following table summarises the mean μ and standard deviation σ of a cohort's marks in English Advanced and Mathematics Advanced.

	Mean	*Standard deviation*
English Advanced mark	57	17
Mathematics Advanced mark	70	7

(a) Let X be the Mathematics Advanced mark of a randomly chosen student, prior to rounding. Suppose that X can be modelled by a normal distribution, with parameters as stated above. **1**

What is the value of $P(X > 70)$?

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(b) Assume that the probability density function of a normal random variable with mean μ and variance σ^2 is **4**

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} .$$

Use the trapezoidal rule with FOUR sub-intervals to estimate $P(49 < X < 77)$ correct to four decimal places. Interpret your conclusions with reference to the empirical rule.

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Question 23 continues on page 21

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Question 23 (continued)

(c) Let Y be the English Advanced mark of a randomly chosen student, prior to rounding. Suppose that Y can be modelled by a normal distribution in a similar way to X. **1**

The probability density functions of X and Y were plotted on the same set of axes. A student recorded the x-coordinate of each function's local maximum.

Which one of the random variables had the lesser value for the local maximum of its probability density function?

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(d) A student scored 76 in both English Advanced and Mathematics Advanced. **2**

Relative to their cohort, which subject did the student perform better in? Justify your answer with appropriate computations.

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(e) Another student attained the same mark for both English Advanced and Mathematics Advanced. She performed equally well relative to both cohorts. **2**

What mark has the student attained for both subjects?

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End of Question 23

Question 24 (7 marks)

The acceleration on a particle P_1 in m s^{-2} is $\frac{d^2x}{dt^2} = e^{-t} + e^{-2t}$ after t seconds. Initially, the particle is $\frac{3}{4}$ m to the right of the origin, travelling at velocity $\frac{dx}{dt} = -\frac{3}{2}$ m s^{-1}.

(a) Show that the displacement of the particle is given by $x = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}$. **2**

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(b) Find the limiting displacement of P_1, and hence state the limiting distance that P_1 travels. **1**

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Question 24 continues on page 23

Question 24 (continued)

Another particle, P_2, moves simultaneously with the first particle. The acceleration P_2 experiences is

$$\frac{d^2x}{dt^2} = -e^{-t} - e^{-2t}.$$

This particle is $\frac{3}{4}$ m to the right of the origin and travelling at a velocity $\frac{dx}{dt} = -\frac{3}{2}$ m s^{-1} when $t = \ln 3$ seconds.

(c) Determine the exact time when P_1 and P_2 are travelling at the same velocity. Do NOT simplify your answer. **4**

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End of Question 24

Question 25 (8 marks)

Consider the function below.

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \le x \le 1, \\ \frac{2}{3} - \frac{1}{6}x & \text{if } 1 < x \le 4, \\ 0 & \text{else} \end{cases}$$

You may assume that $f(x) \ge 0$ for all x in $(-\infty, \infty)$.

(a) Prove the other property required to show that $f(x)$ is a valid density function for a continuous random variable X. **2**

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(b) Sketch $y = f(x)$, showing each point where the rule defining $y = f(x)$ changes. **2**

Question 25 continues on page 25

Question 25 (continued)

(c) State the mode of X. **1**

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(d) Find the median of X. **3**

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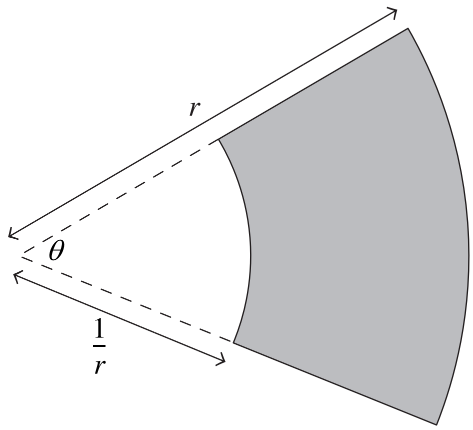
End of Question 25

Question 26 (5 marks)

An annulus sector is made at angle θ such that the inner and outer radii are r metres and $\frac{1}{r}$ metres respectively, as shown in the diagram.



NOT TO SCALE

Assume that the perimeter of this annulus sector is 6 metres.

(a) Show that $\theta = \frac{2(-r^2 + 3r + 1)}{r^2 + 1}$. **2**

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Question 26 continues on page 27

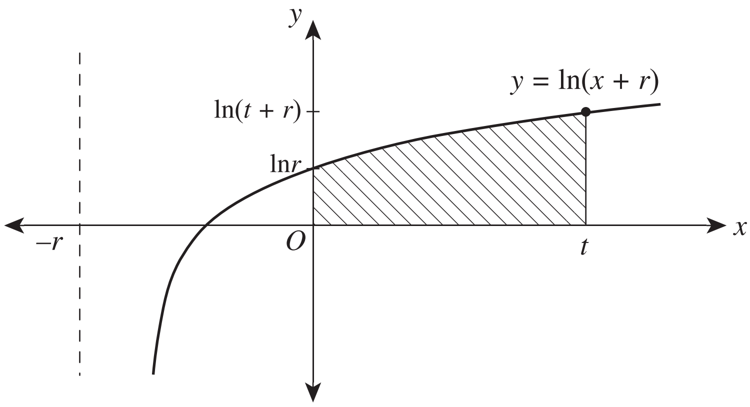
(b) Assume without proof that $-2r^4 + 3r^3 + 3r + 2 = -(2r + 1)(r - 2)(r^2 + 1)$. **3**

If the area of the annulus sector is $A = -r^2 + 3r + 2 - \frac{3}{r} - \frac{1}{r^2}$, find the maximum possible area of the annulus.

End of Question 26

Question 27 (3 marks)

Consider the graph of $y = \ln(x + r)$ shown below. Let $r > 1$. **3**



The region bound by the curve $y = \ln(x + r)$, the coordinate axes, and the line $x = t$ is used as a model for a garden, where $t > 0$.

You may assume without proof that the equation of the curve is equivalently expressed as $x = e^y - r$.

Show that the area of this region, A, in terms of r and t is $A = (t + r)\ln(t + r) - r\ln r - t$.

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Question 28 (5 marks)

It is known that the angles in a triangle sum to π. In a particular triangle, the angles form a geometric progression. One of the angles is $\frac{\pi}{4}$. **5**

Find the TWO possible configurations of the angles of this triangle. Give your answers in simplified, exact form.

End of paper

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi r h$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n - 1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

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Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

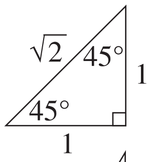
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

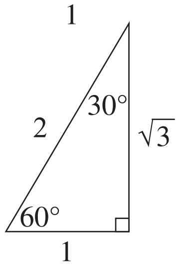
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}[\cos(A - B) + \cos(A + B)]$

$\sin A\sin B = \dfrac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\sin A\cos B = \dfrac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \dfrac{1}{2}[\sin(A + B) - \sin(A - B)]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

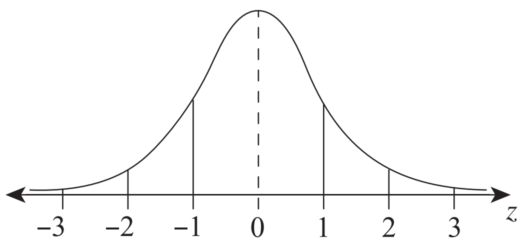
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \displaystyle\int_a^r f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r\, p^r(1 - p)^{n - r}$

$X \sim \text{Bin}(n, p)$
$\Rightarrow P(X = x)$
$= \dbinom{n}{x} p^x(1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\{f(a) + f(b) + 2[f(x_1) + \ldots + f(x_{n-1})]\}$$

where $a = x_0$ and $b = x_n$

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Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^{2} + y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1 x_2 + y_1 y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$

$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$

$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$

Neap

HSC Trial Examination 2020

Mathematics Advanced

DIRECTIONS:

Write your name in the space provided.

Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.

Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark **only one** oval per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: ______________________________

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I
MULTIPLE-CHOICE ANSWER SHEET

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II

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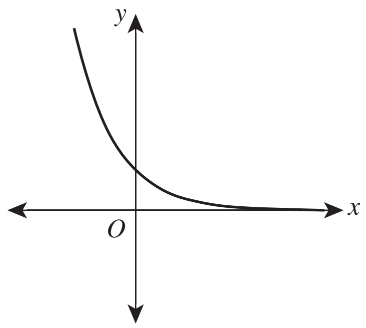
HSC Trial Examination 2020

Mathematics Advanced

Solutions and marking guidelines

Section I

Sample answer	Syllabus content, outcomes and targeted performance bands
Question 1 **B** $\operatorname{cosec}\frac{\pi}{3} = \frac{1}{\sin\frac{\pi}{3}}$ $= \frac{1}{\frac{\sqrt{3}}{2}}$ $= \frac{2}{\sqrt{3}}$ ≈ 1.15	MA-T3 Trigonometric Functions and Graphs MA12–5 Band 2
Question 2 **C** In general, for $f(x) = a\sin(bx + c) + d$, the amplitude is $\|a\|$. Note that $\frac{2\pi}{2} = \pi$ is the period.	MA-T3 Trigonometric Functions and Graphs MA12–5 Band 2
Question 3 **A** From index laws, $f(x) = e^{-x}$. This is a standard, decreasing exponential function, which has a natural domain of all real numbers. 	MA-E1 Logarithms and Exponentials MA12–1 Band 2
Question 4 **A** $y = x^2 - 3$ $\frac{dy}{dx} = 2x$ When $x = 1$: $y = 1^2 - 3$ $= -2$ $\frac{dy}{dx} = -2(1)$ $= -2$ So the tangent has the equation: $y - (-2) = -2(x - (-1))$ $y + 2 = -2x - 2$ $y = -2x - 4$	MA-C1 Introduction to Differentiation MA12–6 Bands 2–3

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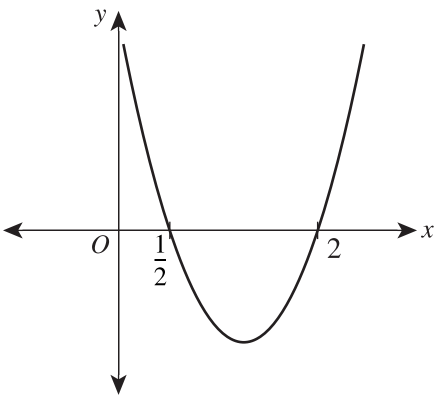
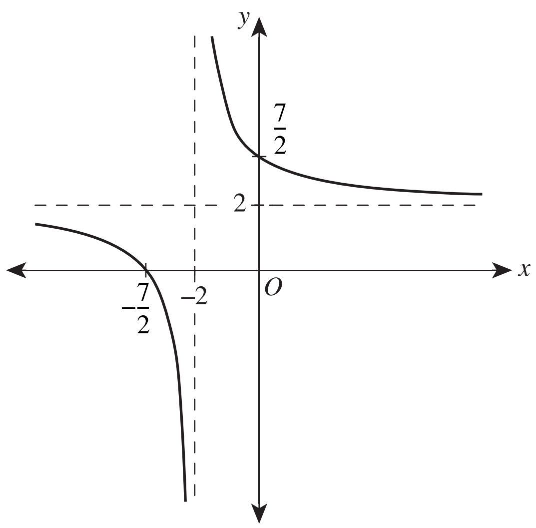
Sample answer	**Syllabus content, outcomes and targeted performance bands**
Question 5 **D** $2x^2 - 5x + 2 \geq 0$ $(2x - 1)(x - 2) \geq 0$  The graph lies above or on the x-axis when $x \leq \frac{1}{2}$ or $x \geq 2$. In interval notation, $x \leq \frac{1}{2}$ is $\left(-\infty, \frac{1}{2}\right]$ and $x \geq 2$ is $[2, \infty)$.	MA-F2 Graphing Techniques MA12–1 Bands 3–4
Question 6 **B** The decreasing trend suggests a negative correlation coefficient, immediately eliminating **C** and **D**. **A** would require almost perfect negative correlation, and thus the points would need to lie almost perfectly on a straight line. The points are spread out in the graph so, by elimination, **B** is the most valid option.	MA-S2 Descriptive Statistics and Bivariate Data Analysis MA12–8 Bands 3–4
Question 7 **C** $f\left(\frac{x}{2}\right)$ dilates outwards from the y-axis by a factor of 2, so the new x-coordinate is 4. This is the only change to the x-coordinate. Then we reflect about the x-axis, which changes the y-coordinate to 3. Finally, we translate downwards by 5 units, so the y-coordinate is now –2.	MA-F2 Graphing Techniques MA12–1 Bands 3–5
Question 8 **D** The result $\int_{-a}^{a} f(x)\, dx = 0$ is true if $f(x)$ is an odd function, but $f(x) = 4x^4 - x^3 + \cos x$ is not odd. Using the reverse chain rule, **A** is true. Considering half the area under a semicircle, **B** is true. Evaluating the reference sheet integral directly, **C** is true.	MA-C4 Integral Calculus MA12–7 Bands 4–5

Sample answer	Syllabus content, outcomes and targeted performance bands
Question 9 **B** $v = 2t - \frac{1}{t+1}$ The particle comes to rest when $v = 0$. $2t - \frac{1}{t+1} = 0$ $2t = \frac{1}{t+1}$ $2t(t+1) = 1$ $2t^2 + 2t - 1 = 0$ $t = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$ $\approx -1.366, 0.366$ As $t \geq 0$, the negative solution does not count. Hence the particle only comes to rest once.	MA-C1 Introduction to Differentiation MA12–3, MA12–6 Bands 4–6
Question 10 **C** Since '$X \geq 1$ and $X \geq 3$' is equivalent to '$X \geq 3$': $P(X \geq 3 \mid X \geq 1) = \frac{P(X \geq 3 \cap X \geq 1)}{P(X \geq 1)}$ $= \frac{P(X \geq 3)}{P(X \geq 1)}$ $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - (0.2 + 0.2(0.8) + 0.2(0.8)^2)$ $= 0.512$ $P(X \geq 1) = 1 - P(X \leq 0)$ $= 1 - 0.2$ $= 0.8$ Hence: $P(X \geq 3 \mid X \geq 1) = \frac{0.512}{0.8}$ $= 0.64$	MA-S1 Probability and Discrete Probability Distributions MA12–8 Bands 5–6

Section II

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 11	
$-x + 4y + 12 = 0$ $4y = x - 12$ $y = \frac{x}{4} - 3$ Therefore: $m = \tan\theta$ $= \frac{1}{4}$ So $\theta = 14°2'$ to the nearest minute.	MA-C1 Introduction to Differentiation MA11–5 Bands 2–3 • Gives the correct solution . . . . 2 • Finds the gradient of the line 1
Question 12	
(a) $\frac{d}{dx}\log_2(\cos x) = \frac{(-\sin x)}{(\ln 2)\cos x}$ $= -\frac{\tan x}{\ln 2}$ *Note: Unsimplified correct responses are awarded full marks.*	MA-C2 Differential Calculus MA12–6 Bands 2–3 • Gives the correct solution 1
(b) $\frac{d}{dx}3^x e^x = 3^x(\ln 3)e^x + 3^x e^x$ $= 3^x e^x(\ln 3 + 1)$ **OR** $\frac{d}{dx}3^x e^x = \frac{d}{dx}(3e)^x$ $= (3e)^x \ln(3e)$ $= 3^x e^x(\ln 3 + 1)$ *Note: Unsimplified correct responses are awarded full marks.*	MA-C2 Differential Calculus MA12–6 Bands 2–3 • Gives the correct solution . . . . 2 • Attempts to use product rule OR uses an index law in preparation for the chain rule.. 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 13	



y-intercept:

$$y = \frac{3}{0+2} + 2$$
$$= \frac{7}{2}$$

x-intercept:

$$\frac{3}{x+2} + 2 = 0$$
$$\frac{3}{x+2} = -2$$
$$-\frac{3}{2} = x + 2$$
$$x = -\frac{7}{2}$$

vertical asymptote:

$$x + 2 = 0$$
$$x = -2$$

horizontal asymptote:

$$y = 0 + 2$$
$$= 2$$

MA-F1 Working with Functions
MA12–1 Bands 2–3

- Sketch demonstrates ALL of:
  - y-intercept
  - x-intercept
  - vertical asymptote
  - horizontal asymptote

AND

- Clearly shows the correct shape 3

- Finds ALL of:
  - y-intercept
  - x-intercept
  - vertical asymptote
  - horizontal asymptote

OR

- Sketch demonstrates TWO of:
  - y-intercept
  - x-intercept
  - vertical asymptote
  - horizontal asymptote 2

- Finds TWO of:
  - y-intercept
  - x-intercept
  - vertical asymptote
  - horizontal asymptote 1

Question 14

(a)

	Male	*Female*	*Total*
Brown	98	**92**	190
Other	**39**	37	76
Total	137	**129**	**266**

MA-S2 Descriptive Statistics and Bivariate Data Analysis
MA12–8 Bands 2–3

- Correctly completes ALL table entries. 2

- Correctly completes at least TWO table entries. 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(b) Using the row for brown eyes: $P(\text{female} \mid \text{brown}) = \frac{92}{190}$ $= \frac{46}{95}$ **OR** Using conditional probability: $P(\text{female} \mid \text{brown}) = \frac{P(\text{female} \cap \text{brown})}{P(\text{brown})}$ $= \frac{\frac{46}{226}}{\frac{190}{266}}$ $= \frac{46}{95}$	MA-S2 Descriptive Statistics and Bivariate Data Analysis MA12–8 Bands 2–3 • Gives the correct solution 1
Question 15	
$\left(1 + \frac{r}{12}\right)^{12} = 1.1127$ $1 + \frac{r}{12} = 1.1127^{\frac{1}{12}}$ $r = 12\left(1.1127^{\frac{1}{12}} - 1\right)$ $\approx 10.73\%$ (correct to two decimal places)	MA-M1 Modelling Financial Situations MA12–2 Bands 2–3 • Gives the correct solution . . . . 2 • Sets up the correct equation 1
Question 16	
$\int_{-5}^{-2} \frac{x^2}{x^3 - 2}\,dx = \frac{1}{3}\int_{-5}^{-2} \frac{x^2}{x^3 - 2}\,dx$ $= \frac{1}{3}\left[\ln\left(\left\lvert x^3 - 2\right\rvert\right)\right]_{-5}^{-2}$ $= \frac{1}{3}(\ln\lvert -10\rvert - \ln\lvert -127\rvert)$ $= \frac{1}{3}\ln\frac{10}{127}$ $= \ln\left(\frac{10}{127}\right)^{\frac{1}{3}}$ Hence $k = \left(\frac{10}{127}\right)^{\frac{1}{3}}$.	MA-C4 Integral Calculus MA12–7 Bands 3–4 • Gives the correct solution . . . . 3 • Correctly evaluates the integral OR equivalent merit. . . . . 2 • Makes progress towards the correct anti-derivative. 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 17	
(a) $x = \pm 2$	MA-F1 Working with Functions MA12–1 Band 2 • Gives the correct solution 1
(b) $f(x) = (x+2)(x-2)^3$ $f'(x) = (x-2)^3 + 3(x+2)(x-2)^2$ $= (x-2)^2[(x-2) + 3(x+2)]$ $= (x-2)^2(4x+4)$ $= 4(x-2)^2(x+1)$ $f''(x) = 4[(x-2)^2 + 2(x-2)(x+1)]$ $= 4(x-2)[(x-2) + 2(x+1)]$ $= 4(x-2)(3x)$ $= 12x(x-2)$	MA-C3 Applications of Differentiation MA12–3, MA12–6 Bands 3–4 • Gives the correct solution . . . . 2 • Correctly proves ONE formula. 1
(c) Setting $f'(x) = 0$ for the stationary points gives $x = 2$ and $x = -1$. $f(2) = (2+2)(2-2)^3 = 0$ $f(-1) = (-1+2)(-1-1)^3 = -27$ Test for concavity change: x: 1, 2, 3 $f''(x)$: –12, 0, 36 *sign*: –, 0, + Since the concavity changes, (2, 0) is a horizontal point of inflection. $f'' = 12(1)(1-2) = -12 < 0$, so (–1, –27) is a local minimum.	MA-C3 Applications of Differentiation MA12–3, MA12–10 Bands 3–4 • Gives the correct solution with test for concavity change at $x = 2$ . . . . 3 • Gives the correct solution . . . . 2 • Gives the coordinates of ALL stationary points 1
(d) For possible points of inflection, set $f''(x) = 0$. $12x(x-2) = 0 \Rightarrow x = 0, 2$ From part (c), (2, 0) is a point of inflection. Test for concavity change at $x = 0$. x: –1, 0, 1 $f''(x)$: 36, 0, –12 *sign*: +, 0, – Concavity does change, and $f(0) = -16$. So (0, –16) is also an inflection point.	MA-C3 Applications of Differentiation MA12–13, MA12–10 Band 3 • Gives correct solution with test for concavity change. . . . . 2 • Finds the x-coordinate of both points of inflection 1

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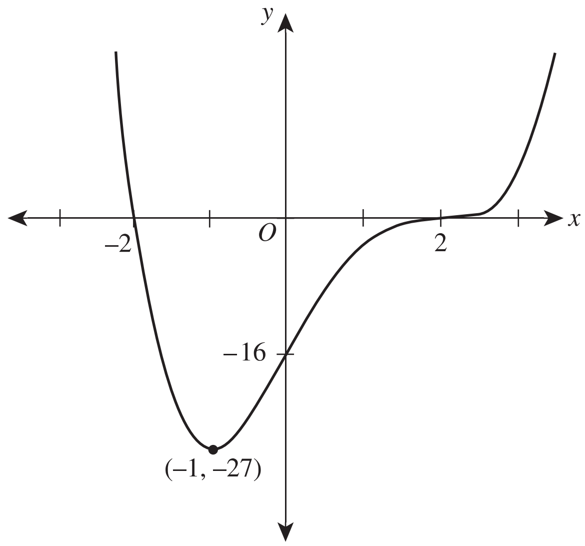
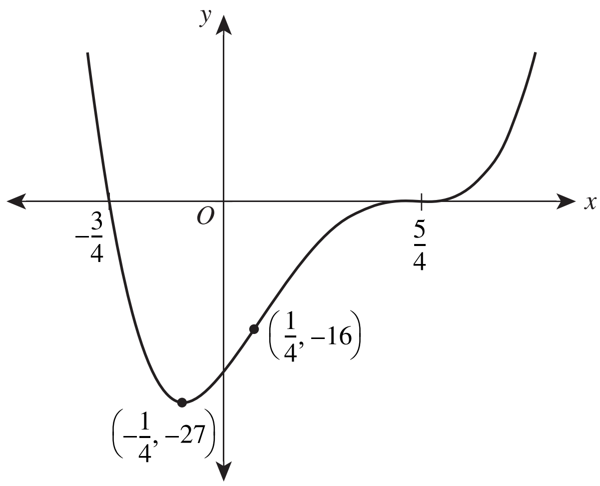
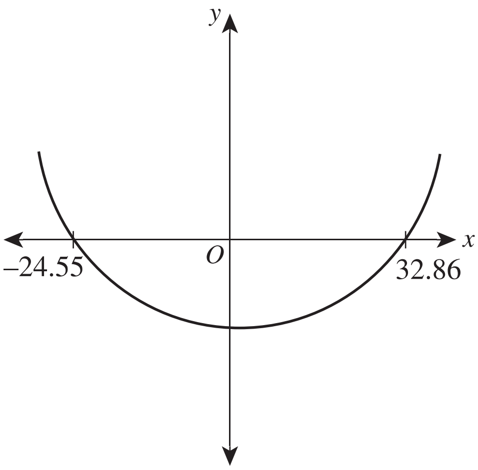
Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(e) 	MA-C3 Applications of Differentiation MA12–1 Bands 3–4 • Gives correct graph. AND • Correctly shows ALL of: • x-intercepts • stationary point • point of inflection 2
	• Gives correct graph. AND • Correctly shows at least ONE of: • x-intercepts • stationary point • point of inflection 1
(f) There is no global maximum. The global minimum is located at (–1, –27).	MA-C3 Applications of Differentiation MA12–1 Bands 2–3 • Gives the correct solution 1
(g) First dilate about the x-axis by a factor of $\frac{1}{2}$. Then translate to the right by $\frac{1}{4}$ units.	MA-F2 Graphing Techniques MA12–1 Bands 3–4 • Gives the correct solution 2
	• Correctly identifies at least ONE transformation in any order 1
(h) 	MA-F2 Graphing Techniques MA12–1 Bands 3–5 • Gives correct graph. AND • Correctly shows ALL of: • x-intercepts • stationary point • point of inflection 3
	• Gives correct graph. AND • Correctly shows TWO of: • x-intercepts • stationary point • point of inflection 2
	• Gives correct graph. AND • Correctly shows ONE of: • x-intercepts • stationary point • point of inflection 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 18	
(a) This is equivalent to a 4 year annuity at rate 4% per annum, with compounding every year. $\therefore FVA = 1.0000 + 1.0400 + 1.0816 + 1.1249$ $= 4.2465$ $PVA = \frac{FVA}{1.04^4}$ $= \frac{4.2465}{1.1699}$ $= 3.6298$ Therefore, to the nearest cent, $PVA = \$3.63$.	MA-M1 Modelling Financial Situations MA12–2 Bands 3–4 • Gives the correct solution . . . . 2 • Identifies the link between the present and future value OR correctly calculates ONLY the future value 1
(b) Let P be the contribution amount. Then $P \times FVA = 1000$. $\therefore P(1.0000 + 1.0800 + 1.1664 + 1.2597 + 1.3605) = 1000$ $P \times 5.8666 = 1000$ $P = \frac{1000}{5.8666}$ $= 170.4564$	MA-M1 Modelling Financial Situations MA12–2 Bands 3–4 • Gives the correct solution . . . . 2 • Correctly calculates the future value of the required annuity OR writes $P \times FVA = 1000$ without attempting to solve. 1
Question 19	
(a) $0 \le x \le 2\pi \Rightarrow 0 \le 2x \le 4\pi$ $\Rightarrow -\frac{\pi}{3} \le 2x - -\frac{\pi}{3} \le \frac{11\pi}{3}$ The solutions to $\tan\left(2x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$ in the interval $\left[-\frac{\pi}{3}, \frac{11\pi}{3}\right]$ are in the first, third, fifth and seventh quadrants: $2x - \frac{\pi}{3} = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi + \frac{\pi}{6}$ $= \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}$ $2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	MA-T3 Trigonometric Functions and Graphs MA12–1, MA12–5 Bands 3–5 • Finds all FOUR solutions for x. . . . . 3 • Finds all FOUR solutions for $2x - \frac{\pi}{3}$ OR finds TWO solutions for x. . . . . 2 • Correct domain adjustment OR finds solutions for $\tan x = \frac{1}{\sqrt{3}}$ 1
(b) *For example:* Consider the asymptote corresponding to $y = \tan x$ at $x = \frac{\pi}{2}$. $2x - \frac{\pi}{3} = \frac{\pi}{2}$ $2x = \frac{5\pi}{6}$ $x = \frac{5\pi}{12}$	MA-T3 Trigonometric Functions and Graphs MA12–1, MA12–5 Band 3 • Gives ANY correct solution 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 20	
(a) $-0.1263 + 10 \times 2.0694 = 20.5677$ mm	MA-S2 Descriptive Statistics and Bivariate Data Analysis MA12–8, MA12–9 Band 2 • Gives the correct solution 1
(b) No. The value $x = 10$ falls outside the range presented in the scatterplot, and hence we performed an extrapolation. Outside the range $1 \le x \le 5$, the linear trend may break on us. In this case, over-watering a plant would likely hinder its growth. *Note: Responses do not require real-world explanations such as over-watering; this information has been included to provide an example of how predictions can be flawed in practice. A correct justification needs only to refer to extrapolation.*	MA-S2 Descriptive Statistics and Bivariate Data Analysis MA12–8, MA12–9, MA12–10 Bands 3–4 • Gives the correct answer. AND • Correctly justifies the answer . . . . 2 • Gives the correct answer 1
Question 21	
$a + 4d = 9$ **(1)** $a + 20d = 425$ **(2)** From **(2)** – **(1)**, $16d = 416 \Rightarrow d = 26$. Subbing into **(1)**, $a + 4 \times 26 = 9 \Rightarrow a = -95$. We require $S_n \ge 10\,487$. That is: $\frac{n}{2}[2(-95) + (n-1)(26)] \ge 10\,487$ $\frac{n}{2}(-190 + 26n - 26) \ge 10\,487$ $13n^2 - 108n - 10\,487 \ge 0$ The solutions to $13n^2 - 108n - 10\,487 = 0$ are: $n = \frac{108 \pm \sqrt{556988}}{26}$ $\approx -24.55, 32.86$  From the graph, since we also require $n \ge 1$, we require n to be larger than 32.86. The smallest value of n is therefore $n = 33$. *Note: Responses do not require a graph.*	MA-F2 Graphing Techniques MA-M1 Modelling Financial Situations MA12–1, MA12-4 Bands 4–5 • Gives the correct solution . . . . 3 • Establishes an inequality of the form $an^2 + bn + c \ge 0$ OR equivalent merit. . . . . 2 • Finds the first term AND finds the common difference 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 22	
(a) interest rate: 1.5% per quarter $A_0 = 4000$ $A_1 = A_0(1.015) + 4000$ $= 4000(1.015) + 4000$ $A_2 = A_1(1.015) + 4000$ $= 4000(1.015)^2 + 4000(1.015) + 4000$ $A_3 = A_2(1.015) + 4000$ $= 4000(1.015)^3 + 4000(1.015^2) + 4000(1.015) + 4000$ Continuing the pattern: $A_n = 4000(1.015)^n + 4000(1.015)^{n-1} + \ldots + 4000(1.015) + 4000$ $= 4000(1 + 1.015 + \ldots + 1.015^{n-1} + 1.015^n)$ $= \dfrac{4000(1.015^{n+1} - 1)}{0.015}$ We require $A_{36} \approx 195\ 940.44$ to the nearest cent.	MA-M1 Modelling Financial Situations MA12–2, MA12–4 Bands 3–4 • Gives the correct solution . . . . 3 • Correctly uses an appropriate geometric series . . . . 2 • Correctly identifies the initial value $A_0 = 4000$ OR the recursive relationship $A_{n+1} = 1.015A_n + 4000$ 1
(b) Let P be the amount required. $P(1.015)^{36} = 195\ 940.44$ $P = \dfrac{195\ 940.44}{1.015^{36}}$ $\approx 114\ 642.740\ 2$ Hence \$114 642.74 to the nearest cent.	MA-M1 Modelling Financial Situations MA12–2 Band 3 • Gives the correct solution 1
(c) $\dfrac{4000(1.015^{n+1} - 1)}{0.015} = 500\ 000$ $1.015^{n+1} - 1 = \dfrac{500\ 000 \times 0.015}{4000}$ $1.015^{n+1} - 1 = 1.875$ $1.015^{n+1} = 2.875$ $(n+1)\ln 1.015 = \ln 2.875$ $n + 1 = \dfrac{\ln 2.875}{\ln 1.015}$ $n = \dfrac{\ln 2.875}{\ln 1.015} - 1$ $= 69.93022\ldots$ Hence Alex's fund will surpass \$500 000 after 70 quarter years. This is equal to 70 × 3 = 210 full months.	MA-E1 Logarithms and Exponentials MA-M1 Modelling Financial Situations MA12–1 Bands 4–5 • Gives the correct solution . . . . 2 • Arrives at an equation in the form $a^k = b$. 1

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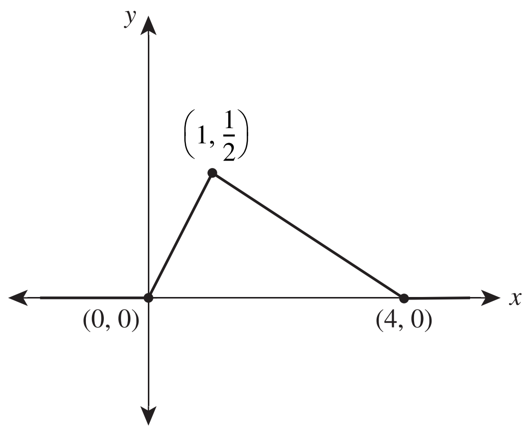
Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 23	
(a) $P(X > 70) = \frac{1}{2}$	MA-S3 Random Variables MA12–8, MA12–9 Band 4 • Gives the correct solution 1
(b) $P(49 \le X \le 77) = \int_{49}^{77} \frac{1}{\sqrt{2\pi(49)}} e^{-\frac{(x-70)^2}{2(49)}} dx$ $= \frac{1}{\sqrt{98\pi}} \int_{49}^{77} e^{-\frac{(x-70)^2}{98}} dx$ $\approx \frac{1}{\sqrt{98\pi}} \frac{77-49}{2 \times 4}\left[e^{-\frac{(49-70)^2}{98}} + 2\left(e^{-\frac{(56-70)^2}{98}} + e^{-\frac{(63-70)^2}{98}} + e^{-\frac{(70-70)^2}{98}}\right) + e^{-\frac{(77-70)^2}{98}}\right]$ $= \frac{1}{2\sqrt{2\pi}}\left[e^{-\frac{9}{2}} + 2\left(e^{-2} + e^{-\frac{1}{2}} + 1\right) + e^{-\frac{1}{2}}\right]$ ≈ 0.8181 (to four decimal places) The probability required is the probability that a mark lies within three standard deviations to the left of the mean, and one standard deviations to the right of the mean. From the empirical rule, we know that approximately $\frac{99.7\%}{2} + \frac{68\%}{2} = 83.85\%$ of all marks lie in this range. Our approximation reflects this well, noting that we have only used four sub-intervals and hence cannot expect an extremely precise answer. *Note: This question is more easily handled by first standardising the required probability. That is, set* $Z = \frac{X-70}{7}$ *and observe that* $Z \sim N(0, 1)$. *Accept responses that correctly use this approach.*	MA-C4 Integral Calculus MA-S3 Random Variables MA12–7, 8, 9, 10 Bands 4–6 • Gives correct approximation. AND • Gives a valid interpretation . . . . 4 • Gives correct approximation. OR • Gives a valid interpretation . . . . 3 • Appropriately uses the trapezoidal rule. OR • States integral AND links to empirical rule . . . . 2 • States the required integral 1
(c) Y *Note: The local maximum of a normal probability density function occurs at its mean.*	MA-S3 Random Variables MA12–8 Band 3 • Gives the correct solution 1
(d) $z_{\text{English}} = \frac{76-57}{17}$ $= 1.11764$ $z_{\text{Mathematics}} = \frac{76-70}{7}$ $= 0.85714$ Comparing these z-scores, the student performed better relative to their English cohort.	MA-S3 Random Variables MA12–8, MA12–9 Band 3 • Gives the correct answer AND computes TWO z-scores . . . . 2 • Computes ONE z-scores 1

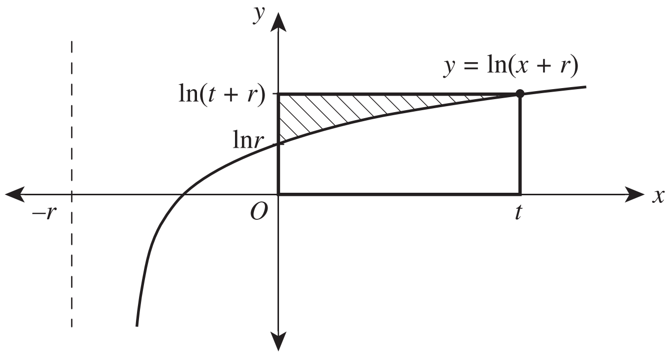
Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(e) Let x be the student's mark. $\frac{x-57}{17}=\frac{x-70}{7}$ $7(x-57)=17(x-70)$ $7x-399=17x-1190$ $791=10x$ $x=79.1$	MA-S3 Random Variables MA12–8, MA12–9, MA12–10 Band 4 • Gives the correct solution . . . . 2 • Sets up the correct equation 1
Question 24	
(a) $\frac{d^2x}{dt^2}=e^{-t}+e^{-2t}$ $\frac{dx}{dt}=-e^{-t}-\frac{1}{2}e^{-2t}+c_1$ When $t=0$, $\frac{dx}{dt}=-\frac{3}{2}$. $\therefore -\frac{3}{2}=-1-\frac{1}{2}+c_1 \Rightarrow c_1=0$ $\therefore \frac{dx}{dt}=-e^{-t}-\frac{1}{2}e^{-2t}$ $x=e^{-t}+\frac{1}{4}e^{-2t}+c_2$ When $t=0$, $x=\frac{3}{4}$. $\therefore \frac{3}{4}=1+\frac{1}{4}+c_2 \Rightarrow c_2=-\frac{1}{2}$ $\therefore x=e^{-t}+\frac{1}{4}e^{-2t}-\frac{1}{2}$ as required.	MA-C3 Applications of Differentiation MA12–3 Bands 3–4 • Gives the correct solution . . . . 2 • Correct expression for the velocity OR gives TWO correct anti-derivatives despite incorrect constants of integration. 1
(b) As $t\to\infty$, $e^{-t}\to 0$ and $e^{-2t}\to 0$. Hence $x\to -\frac{1}{2}$; that is, the limiting displacement is $x=-\frac{1}{2}$ metres. Therefore the limiting distance travelled is $\frac{3}{4}-\left(-\frac{1}{2}\right)=\frac{5}{4}$ metres.	MA-E1 Logarithms and Exponentials MA-C3 Applications of Differentiation MA12–1 Bands 3–4 • Gives the correct solution 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(c) For P_2, $\frac{d^2x}{dt^2} = e^{-t} + \frac{1}{2}e^{-2t} + c_3$. When $t = \ln 3$, $\frac{dx}{dx} = -\frac{3}{2}$. $\therefore -\frac{3}{2} = e^{-\ln 3} + \frac{1}{2}e^{-2\ln 3} + c_3$ $= e^{\ln(3^{-1})} + \frac{1}{2}e^{\ln(3^{-2})} + c_3$ $= 3^{-1} + \frac{1}{2}(3^{-2}) + c_3$ $c_3 = -\frac{17}{9}$ Hence we require: $-e^{-t} - \frac{1}{2}e^{-2t} = e^{-t} + \frac{1}{2}e^{-2t} - \frac{17}{9}$ $0 = e^{-2t} + 2e^{-t} - \frac{17}{9}$ $0 = 9(e^{-t})^2 + 18e^{-t} - 17$ $\therefore e^{-t} = \frac{-18 \pm \sqrt{18^2 - 4(9) - 17}}{2(9)}$ $e^{-t} = \frac{-18 \pm \sqrt{936}}{18}$ As $e^{-t} > 0$ for all t, the only candidate solution is: $e^{-t} = \frac{-18 + \sqrt{936}}{18}$ $t = -\ln\left(\frac{-18 \pm \sqrt{936}}{18}\right)$ Hence the particles move at the same velocity at $-\ln\left(\frac{-18 \pm \sqrt{936}}{18}\right)$ seconds.	MA-E1 Logarithms and Exponentials MA-C3 Applications of Differentiation MA12–3, MA12–10 Bands 4–6 • Gives the correct solution . . . . 4 • Makes worthwhile progress solving a relevant equation . . . . 3 • Gives correct expression for $\frac{dx}{dt}$ . . . . 2 • Gives correct anti-derivative 1
Question 25	
(a) Note that the range of values of X is $0 \le x \le 4$. However, the rule defining the density function of X changes at $x = 1$, so we consider the regions $0 \le x \le 1$ and $1 \le x \le 4$ separately. $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 \frac{x}{2}dx + \int_1^4 \frac{2}{3} - \frac{x}{6}dx$ $= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{2x}{3} - \frac{x^2}{12}\right]_1^4$ $= \left(\frac{1}{4} - 0\right) + \left[\left(\frac{8}{3} - \frac{1}{12}\right) - \left(\frac{2}{3} - \frac{1}{12}\right)\right]$ $= 1$	MA-S3 Random Variables MA12–7, MA12–8 Band 4 • Gives correct solution . . . . 2 • Identifies both of the integrals that need to be computed 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(b)  *Note: As f(x) is defined by a linear function on each interval, it suffices to find the endpoints of each interval and sketch a straight line through the endpoints. For example, along* $1 < x < 4$, *we observe that* $\frac{2}{3} - \frac{1}{6} \times 1 = \frac{1}{2}$, and $\frac{2}{3} - \frac{1}{6} \times 4 = 0$. *Hence we draw a straight line joining* $\left(1, \frac{1}{2}\right)$ *and* $(4, 0)$ *for this part of the graph.*	MA-F1 Working with Functions MA12–1 Bands 4–5 • Gives correct graph. AND • Labels required points . . . . 2 • Gives correct graph. OR • Labels required points 1
(c) The mode of X is 1. *Note: According to the graph, the global maximum of* $y = f(x)$ *is at the peak of the triangle-like shape, which is* $x = 1$.	MA-S3 Random Variables MA12–8 Band 3 • Gives correct solution 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
(d) Let $F(x)$ be the CDF of X. Using the computations in part (a): $F(1) = \int_0^1 \frac{1}{2}t\,dt$ $= \frac{1}{4}$ Since the median is the solution to $F(x) = \frac{1}{2}$, and the CDF is an increasing function, the median must therefore be in the interval [1, 4]. For all x in [1, 4]: $F(x) = \frac{1}{4} + \int_0^1 \frac{2}{3} - \frac{t}{6}dt$ $= \frac{1}{4} + \left[\frac{2t}{3} - \frac{t^2}{12}\right]_1^x$ $= \frac{1}{4} + \left(\frac{2x}{3} - \frac{x^2}{12}\right) - \left(\frac{2}{3} - \frac{1}{12}\right)$ $= -\frac{x^2}{12} + \frac{2x}{3} - \frac{1}{3}$ Hence we require: $-\frac{x^2}{12} + \frac{2x}{3} - \frac{1}{3} = \frac{1}{2}$ $-\frac{x^2}{12} + \frac{2x}{3} - \frac{5}{6} = 0$ $x^2 - 8x + 10 = 0$ $(x-4)^2 = 6$ $x = 4 \pm \sqrt{6}$ Since we require $1 \le x \le 4$, the median must be at $x = 4 - \sqrt{6}$.	MA-S3 Random Variables MA12–8 Bands 4–6 • Gives correct solution . . . . 3 • Obtains the CDF of X in the interval [1, 4] AND justifies the need for it . . . . . 2 • Recognises the interval that the median is in OR any worthwhile effort in computing the CDF 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 26	
(a) Both straight edges have the length $r - \frac{1}{r}$, and the arc lengths of the sectors are $r\theta$ and $\frac{1}{r}\theta$. $\therefore 6 = r\theta + \frac{1}{r}\theta + 2\left(r - \frac{1}{r}\right)$ $\theta\left(r + \frac{1}{r}\right) = 6 - 2\left(r - \frac{1}{r}\right)$ $= 2\left(3 - r + \frac{1}{r}\right)$ $\theta = \dfrac{2\left(3 - r + \frac{1}{r}\right)}{r + \frac{1}{r}}$ $= \dfrac{2(-r^2 + 3r + 1)}{r^2 + 1}$	MA-T1 Trigonometry and Measure of Angles MA11–3, MA12–1 Bands 4–5 • Gives correct solution . . . . . 2 • Gives correct expression for the perimeter 1
(b) $\dfrac{dA}{dr} = -2r + 3 + \dfrac{3}{r^2} + \dfrac{2}{r^3}$ $= \dfrac{-2r^4 + 3r^3 + 3r + 2}{r^4}$ $= \dfrac{-(2r+1)(r-2)(r^2+1)}{r^4}$ Setting $\dfrac{dA}{dr} = 0$ to maximise A, we see that the only solution satisfying $r \geq 1$ is $r = 2$. Differentiating $\dfrac{dA}{dr} = -2r + 3 + \dfrac{3}{r^2} + \dfrac{2}{r^3}$ gives $\dfrac{d^2A}{dr^2} = -2 - \dfrac{6}{r^3} + \dfrac{6}{r^4}$. When $r = 2$, $\dfrac{d^2A}{dr^2} = -\dfrac{25}{8} < 0$, so $r = 2$ gives the maximum area. Therefore the maximum area is $-(2)^2 + 3(2) + 2 - \dfrac{3}{2} - \dfrac{1}{2^2}\text{ m}^2 = \dfrac{9}{4}\text{ m}^2$.	MA-C3 Applications of Differentiation MA12–3, MA12–6, MA12–10 Band 4–5 • Gives correct solution . . . . . 3 • Makes worthwhile progress finding the maximum . . . . . 2 • Differentiates given expression for A 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 27	
Consider an area with respect to the y-axis: $e^y = x + r$ $x = e^y - r$  Comparing areas: $A = t \times \ln(t+r) - \int_{\ln r}^{\ln(t+r)} e^y - r\,dy$ $= t\ln(t+r) - [e^y - ry]_{\ln r}^{\ln(t+r)}$ $= t\ln(t+r) - ((t+r) - r\ln(t+r)) + (r - r\ln r)$ [noting $e^{\ln x} = x$] $= t\ln(t+r) - (t+r) - r\ln(t+r) + r - r\ln r$ $= (t+r)\ln(t+r) - r\ln r - t$	MA-C4 Integral Calculus MA12–7, MA12–9, MA12–10 Bands 4–6 • Gives correct solution . . . . 3 • Correctly compares areas, including the correct y-intercept . . . . 2 • Switches x to be the subject. 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 28	
Case 1: If $\frac{\pi}{4}$ is the smallest or largest angle: Let the angles be $\frac{\pi}{4}, \frac{\pi r}{4}, \frac{\pi r^2}{4}$. $\frac{\pi}{4} + \frac{\pi r}{4} + \frac{\pi r^2}{4} = \pi$ $1 + r + r^2 = 4$ $r^2 + r - 3 = 0$ $r = \frac{-1 \pm \sqrt{1^2 - 4(1)}}{2(1)}$ $= \frac{-1 \pm \sqrt{13}}{2}$ We must have $r > 0$ or else one angle will be negative. Hence $r = \frac{\sqrt{13} - 1}{2}$, and: $r^2 = \left(\frac{\sqrt{13} - 1}{2}\right)^2$ $= \frac{13 - 2\sqrt{13} + 1}{4}$ $= \frac{7 - \sqrt{13}}{2}$ Hence one configuration of the angles is $\frac{\pi}{4}, \frac{\pi(\sqrt{13} - 1)}{8}, \frac{\pi(7 - \sqrt{13})}{8}$. **Case 2:** If $\frac{\pi}{4}$ is the middle angle: Let the angles be $\frac{\pi}{4r}, \frac{\pi}{4}, \frac{\pi r}{4}$. $\frac{\pi}{4r} + \frac{\pi}{4} + \frac{\pi r}{4} = \pi$ $\frac{1}{r} + 1 + r = 4$ $r - 3 + \frac{1}{r} = 0$ $r^2 - 3r + 1 = 0$ $r = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ $= \frac{3 \pm \sqrt{5}}{2}$	MA-M1 Modelling Financial Situations MA12–1, MA12–4, MA12–10 Band 6 • Gives correct solution in simplified, exact form . . . . . 5 • Gives correct solution . . . . . 4 • Identifies the TWO possible cases AND successfully finds ONE configuration . . . . . 3 • Identifies the TWO possible cases OR successfully finds ONE configuration . . . . . 2 • Identifies ONE possible case 1

Sample answer	Syllabus content, outcomes and targeted performance bands and marking guide
Question 28 (continued)	
Note that: $\left(\frac{3+\sqrt{5}}{2}\right)^{-1} = \frac{2}{3+\sqrt{5}}$ $= \frac{2(3-\sqrt{5})}{9-5}$ $= \frac{3-\sqrt{5}}{2}$ Hence both values of *r* obtained above give the same second configuration of angles: $\frac{\pi(3-\sqrt{5})}{8}, \frac{\pi}{4}, \frac{\pi(3+\sqrt{5})}{8}$	