CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 1 - MARKING GUIDELINES

Section I

10 Marks
Multiple-choice Answer Key

Question	Answer
1	B
2	A
3	A
4	D
5	B
6	A
7	C
8	C
9	B
10	D

Question 1 (1 mark)
Outcomes Assessed: ME11-5
Targeted Performance Bands: E2

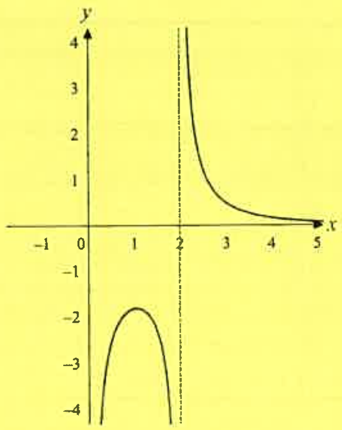
Solution	Mark
Using the pigeonhole principle. $\frac{110}{12} = 9\frac{1}{6}$ $\therefore$ if there are 110 students, the minimum number of students that have a birthday in at least one month is 10. Hence (B).	1

6200-2

Question 2 (1 mark)
Outcomes Assessed: ME11-2
Targeted Performance Bands: E2

Solution	Mark
If a polynomial $P(x)$ has a triple root at $x=-2$ it can be written as: $P(x)=(x+2)^3Q(x)$ $P'(x)=3(x+2)^2Q(x)+(x+2)^3Q'(x)$ $=(x+2)^2\left[3Q(x)+(x+2)Q'(x)\right]$ $\therefore\ (x+2)^2$ is a factor of $P'(x)$ Hence (A).	1

Question 3 (1 mark)
Outcomes Assessed: ME11-1
Targeted Performance Bands: E2-E3

Solution	Mark
at $x=0^+$, $\dfrac{1}{f(x)}\to-\infty$ at $x=2^-$, $\dfrac{1}{f(x)}\to-\infty$ at $x=2^+$, $\dfrac{1}{f(x)}\to\infty$ at $x=1$, $\dfrac{1}{f(x)}=-2$ Hence (A)  	1

Question 4 (1 mark)
Outcomes Assessed: ME11-3
Targeted Performance Bands: E2-E3

Solution	Mark
let $t = \tan\frac{\theta}{2}$ $\therefore \cos\theta = \frac{1-t^2}{1+t^2}$ and $\sin\theta = \frac{2t}{1+t^2}$ $\frac{\cos\theta - 1}{2\sin\theta} = \frac{\frac{1-t^2}{1+t^2} - 1}{2\times\frac{2t}{1+t^2}}$ $= \frac{1-t^2-(1+t^2)}{4t}$ $= \frac{-2t^2}{4t}$ $= \frac{-t}{2}$ Hence (D)	1

Question 5 (1 mark)
Outcomes Assessed: ME11-5
Targeted Performance Bands: E2-E3

Solution	Mark
$n(\text{two Os separated}) = n(\text{unrestricted}) - n(\text{two Os together})$ $= \frac{6!}{2!} - 5!$ $= 240$ ways Hence (B)	1

Question 6 (1 mark)
Outcomes Assessed: ME12-5
Targeted Performance Bands: E3

Solution	Mark
The variance of a Bernoulli distribution is given by: $\sigma^2 = p(1-p)$ $= 0.8(1-0.8)$ $= 0.16$ Hence (A)	1

Question 7 (1 mark)
Outcomes Assessed: ME12-4
Targeted Performance Bands: E3

Solution	Mark
$\cos^2 x - \sin^2 x = \cos 2x$ $\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$ $= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + c$ $= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ $\therefore \int (\sin^2 x + x^2)\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{x^3}{3} + c$ Hence (C)	1

Question 8 (1 mark)
Outcomes Assessed: ME12-2
Targeted Performance Bands: E3

Solution	Mark
$\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j},\ \underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert^2}\underset{\sim}{a}$ $= \frac{2\times 3 + (-5\times 4)}{\left(\sqrt{2^2+5^2}\right)^2}\left[2\underset{\sim}{i} - 5\underset{\sim}{j}\right]$ $= \frac{-14}{29}\left[2\underset{\sim}{i} - 5\underset{\sim}{j}\right]$ Hence (C)	1

Question 9 (1 mark)
Outcomes Assessed: ME11-3
Targeted Performance Bands: E3-E4

Solution	Mark
The graph of $y = \cos^{-1} x + 1$ is dilated vertically by a scale factor of 2 $y = 2(\cos^{-1} x + 1)$ $= 2\cos^{-1} x + 2$ The graph of $y = 2\cos^{-1} x + 2$ is then shifted up 3 units $y = 2\cos^{-1} x + 2 + 3$ $= 2\cos^{-1} x + 5$ Hence (B)	1

Question 10 (1 mark)
Outcomes Assessed: ME12-4
Targeted Performance Bands: E3-E4

Solution	Mark
The slope field follows the following pattern: as $x \to -\infty, \frac{dy}{dx} \to \infty$ for $x \in (-\infty, \infty), \frac{dy}{dx} > 0$ and decreasing as $x \to \infty, \frac{dy}{dx} \to 0$ $\therefore \frac{dy}{dx} = e^{-x}$ Hence (D)	1

Section II
60 marks

Question 11 (15 marks)

11 (a) (3 marks)
Outcomes assessed: ME11-2
Targeted Performance Bands: E2

Criteria	Marks
• Correct solution	3
• Multiplies both sides of the inequality by $(x+3)^2$ or equivalent merit	2
• Recognises that $x=-3$ cannot be part of solution	1

Sample Answer:

$\frac{2x}{x+3} \le 1$ Note $x \ne -3$

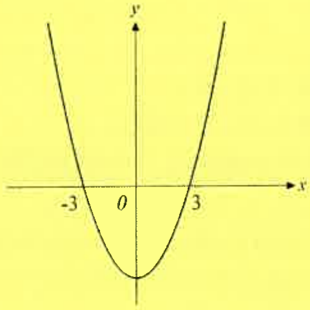
$2x(x+3) \le (x+3)^2$

$2x(x+3)-(x+3)^2 \le 0$

$(x+3)\big(2x-(x+3)\big) \le 0$

$(x+3)(x-3) \le 0$

From graph, solution is $-3 < x \le 3$.



11 (b) (2 marks)
Outcomes Assessed: ME11-2
Targeted Performance Bands: E2

Criteria	Marks
• Correct solution	2
• Attempts to use the remainder theorem	1

Sample Answer:

$P(x) = 2x^3 + kx^2 - 1$

Given $P(-2) = 7$

$7 = 2(-2)^3 + k(-2)^2 - 1$

$7 = -17 + 4k$

$\therefore k = 6$

11 (c) (3 marks)
Outcomes Assessed: ME12-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	3
• Two correct binomial probabilities	2
• Attempted use of binomial probability	1

Sample Answer:

Standard Room: $X \sim B(6, 0.064)$

$$P(X=2) = {}^{6}C_{2}(0.064)^{2}(0.936)^{4}$$
$$= 0.0471 \quad (4 \text{ dp})$$

Executive Room: $X \sim B(5, 0.131)$

$$P(X=2) = {}^{5}C_{2}(0.131)^{2}(0.869)^{3}$$
$$= 0.1126 \quad (4 \text{ dp})$$

$$P(2 \text{ Standard and } 2 \text{ Executive unoccupied}) = 0.0471 \times 0.1126$$
$$= 0.0053 \quad (4 \text{ dp})$$

11 (d) (i) (1 mark)
Outcomes Assessed: ME11-5
Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct solution	1

Sample Answer:

${}^{10}C_{5} = 252$

$\therefore$ there are 252 possible committees that can be formed with no restrictions.

11 (d)(ii) (2 marks)

Outcomes Assessed: ME11-5

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Makes some progress towards solution	1

Sample Answer:

Majority of women can be formed with 5 women, 4 women or 3 women.

$^6C_4 \times {}^4C_1 = 60$

$^6C_3 \times {}^4C_2 = 120$

$^6C_5 \times {}^4C_0 = 6$

$\therefore$ there are 186 possible committees that can be formed if the majority of members are women.

11 (e) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Correct magnitude or direction	1

Sample Answer:

$$\left|\overrightarrow{AB}\right| = \sqrt{(3-2)^2 + (-1-1)^2}$$

$$= \sqrt{5}$$

$$\tan\theta = \frac{-1-1}{3-1}$$

$$\theta = \tan^{-1}(-2)$$

$$= 297° \text{ (nearest degree)}$$

11 (f) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Attempts to use the dot product to calculate the value of x	1

Sample Answer:

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular then $\underset{\sim}{a}.\underset{\sim}{b}=0$

$2\times(-3)+(3)\times x=0$

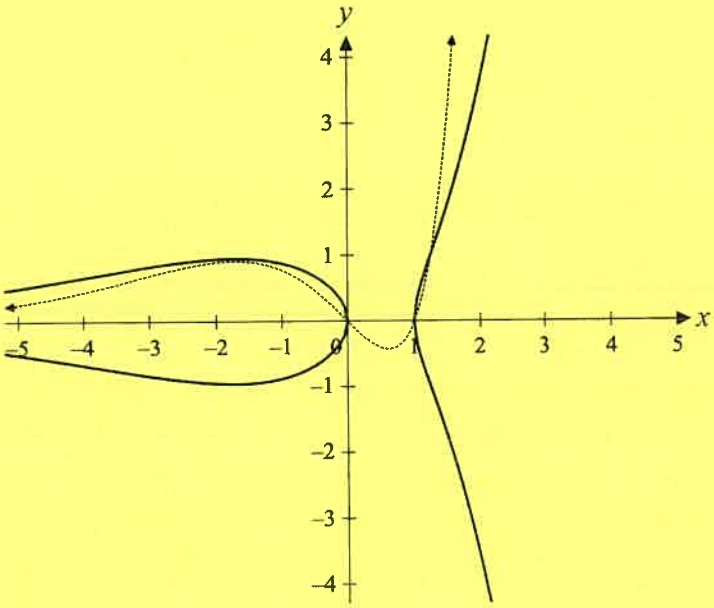
$\therefore x=2$

Question 12 (15 marks)

12 (a)(i) (2 marks)
Outcomes assessed: ME11-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Shows some features of the graph of $y^2 = f\left(x\right)$ or the correct graph of $y = \sqrt{f\left(x\right)}$	1

Sample Answer:



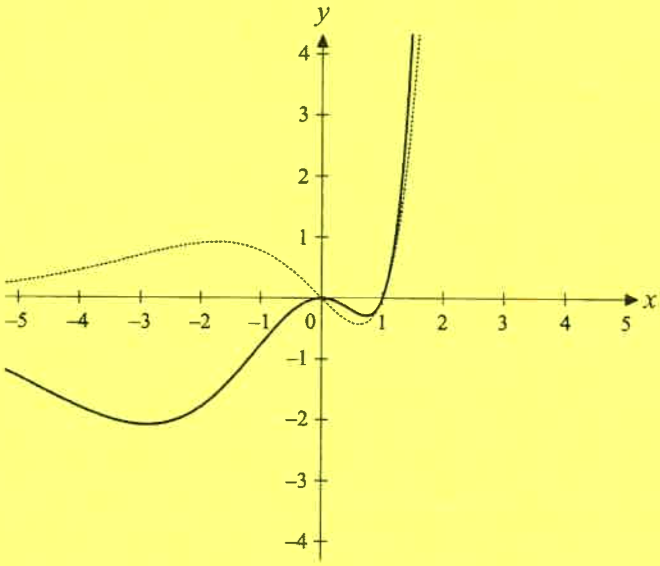
12 (a)(ii) (2 marks)

Outcomes assessed: ME11-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Shows some features of the graph of $y = x\,f\left(x\right)$	1

Sample Answer:



12 (b) (3 marks)

Outcomes Assessed: ME12-3

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Finds correct values of x outside of the specified domain	2
• Forms a correct equation using the auxiliary method	1

Sample Answer:

$$\sin x+\sqrt{3}\cos x=2\sin\left(x+\frac{\pi}{3}\right)$$

$\therefore \sin x+\sqrt{3}\cos x=1$ becomes

$$2\sin\left(x+\frac{\pi}{3}\right)=1$$

$$\sin\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$$

$$\therefore x=\frac{\pi}{2},\ \frac{11\pi}{6}\ \text{for}\ x\in\left[0,2\pi\right]$$

12 (c) (3 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Correct integration	2
• Attempts to form an integral involving $\tan^{-1} x$	1

Sample Answer:

$$\int_0^{\frac{4}{3}} \frac{dx}{16+9x^2}$$

$$= \frac{1}{3}\int_0^{\frac{4}{3}} \frac{3dx}{16+(3x)^2}$$

$$= \frac{1}{4} \times \frac{1}{3}\left[\tan^{-1}\frac{3x}{4}\right]_0^{\frac{4}{3}}$$

$$= \frac{1}{12}\left[\tan^{-1}\left(\frac{3}{4}\times\frac{4}{3}\right) - \tan^{-1}(0)\right]$$

$$= \frac{1}{12}\tan^{-1} 1$$

$$= \frac{\pi}{48}$$

12 (d) (3 marks)

Outcomes Assessed: ME12-4

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Correct substitution and change of variable or equivalent merit	2
• Forms correct integral	1

Sample Answer:

$$V = \pi\int_0^1 y^2\ dx$$

$$= \pi\int_0^1 x^2\left(x^3-3\right)^6\ dx$$

$$= \pi\int_0^1 x^2\left(x^3-3\right)^6\ dx$$

$$= \pi\int_{-3}^{-2} u^6.\frac{1}{3}du$$

$$= \frac{\pi}{3}\left[\frac{u^7}{7}\right]_{-3}^{-2}$$

$$= \frac{\pi}{3}\left[\frac{2059}{7}\right]$$

$$= \frac{2059\pi}{21}\ \text{units}^3$$

$u = x^3 - 3$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3}du = x^2 dx$

$x = 1, u = -2$

$x = 0, u = -3$

12 (e) (2 marks)
Outcomes Assessed: ME11-4
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Obtains expression for $\frac{dP}{dt}$	1

Sample Answer:

$$\frac{dV}{dt}=100$$

$$\frac{dP}{dt}=\frac{dV}{dt}\times\frac{dP}{dV}$$

$$PV=45000 \to P=\frac{45000}{V}$$

$$\frac{dP}{dV}=\frac{-45000}{V^2}$$

$$\therefore \frac{dP}{dt}=100\times\frac{-45000}{V^2}$$

When $V=4000$, $\frac{dP}{dt}=100\times\frac{-45000}{4000^2}$
$=-0.28$

$\therefore$ the air pressure is decreasing at a rate of 0.28 $\text{g/cm}^2\text{/s}$.

Question 13 (15 marks)

13 (a) (2 marks)
Outcomes assessed: ME11-2
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Attempts to eliminate parameter by squaring both equations	1

Sample Answer:

$x = 1 + 2\cos 2t$ and $y = 2 + 2\sin 2t$

$x - 1 = 2\cos 2t \qquad (1)$

$y - 2 = 2\sin 2t \qquad (2)$

$(1)^2 + (2)^2$ gives:

$$\begin{aligned}(x-1)^2 + (y-2)^2 &= 4\cos^2 2t + 4\sin^2 2t \\ &= 4(\cos^2 2t + \sin^2 2t) \\ &= 4\end{aligned}$$

$\therefore$ the Cartesian equation is $(x-1)^2 + (y-2)^2 = 4$

13 (b)(i) (3 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E4

Criteria	Marks
• Correct solution	3
• Correct integration to displacement	2
• Correct components of velocity	1

Sample Answer:

Find the components of displacement

$\ddot{y} = -g$

$\dot{y} = -gt + c_1$

when $t = 0$, $\dot{y} = \dfrac{15\sqrt{2}}{2}$ $\quad \therefore c_1 = \dfrac{15\sqrt{2}}{2}$

$\therefore \dot{y} = -gt + \dfrac{15\sqrt{2}}{2}$

$y = -\dfrac{gt^2}{2} + \dfrac{15\sqrt{2}}{2}t + c_2$

when $t = 0$, $y = 30$ $\therefore c_2 = 30$

$\therefore y = -\dfrac{gt^2}{2} + \dfrac{15\sqrt{2}}{2}t + 30$

$\therefore y = -5t^2 + \dfrac{15\sqrt{2}}{2}t + 30 \quad$ using $g = 10$

$\therefore \underset{\sim}{s} = \left(\dfrac{15\sqrt{2}}{2}t\right)\underset{\sim}{i} + \left(-5t^2 + \dfrac{15\sqrt{2}}{2}t + 30\right)\underset{\sim}{j}$

$\ddot{x} = 0$

$\dot{x} = c_3$

when $t = 0$, $\dot{x} = \dfrac{15\sqrt{2}}{2}$ $\quad \therefore c_3 = \dfrac{15\sqrt{2}}{2}$

$\therefore \dot{x} = \dfrac{15\sqrt{2}}{2}$

$x = \dfrac{15\sqrt{2}}{2}t + c_4$

when $t = 0$, $x = 0$ $\therefore c_4 = 0$

$\therefore x = \dfrac{15\sqrt{2}}{2}t$

13 (b)(ii) (1 mark)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

For the ball to reach the ground the component of $\underset{\sim}{j}$ must equal zero.

$$-5t^2+\frac{15\sqrt{2}}{2}t+30=0$$

$$t=\frac{-\frac{15\sqrt{2}}{2}\pm\sqrt{\left(-\frac{15\sqrt{2}}{2}\right)^2-4(-5)(30)}}{2(-5)}$$

$$=\frac{-\frac{15\sqrt{2}}{2}\pm\sqrt{712.5}}{-10}$$

$$=3.7\text{ seconds }(1\text{ dp})\qquad\text{taking } t>0$$

13 (c)(i) (2 marks)
Outcomes Assessed: ME11-1
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Correct equation of f^{-1} without specifying domain	1

Sample Answer:

$$f \circ g = \sqrt{2(x+2)-1}$$
$$= \sqrt{2x+3}$$

$(f \circ g)^{-1}$ is given by rearranging:

$$x = \sqrt{2y+3}$$
$$\therefore y = \frac{x^2-3}{2}, \text{ for } x \geq 0$$

13 (c)(ii) (2 marks)

Outcomes Assessed: ME11-1

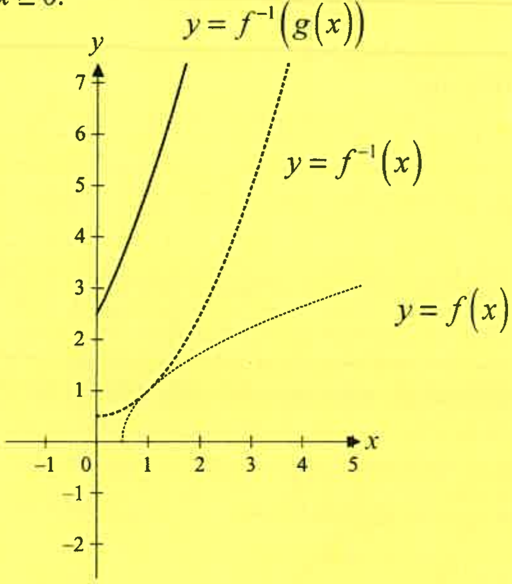
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Correct graph without correct domain	1

Sample Answer:

The domain of $f^{-1}(x)$ is $x \geq 0$

$\therefore$ the domain of $f^{-1}(g(x))$ is also $x \geq 0$.



13 (d) (3 marks)
Outcomes Assessed: ME12-1
Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correctly sets up the statement that needs to be proved	2
• Shows that $P(n)$ is true for $n=1$	1

Sample Answer:

Let $P(n)$ be the proposition that $7^n - 3^n$ is divisible by 4 for $n \geq 1$.

When $n=1$, LHS $= 7^1 - 3^1 = 4$

$$= 4 \times 1$$

$\therefore P(n)$ is true when $n=1$.

Assume that $P(n)$ is true when $n=k$.

i.e. assume that $7^k - 3^k = 4M$, where $M \in \mathbb{Z}^+$

Required to prove that $P(n)$ is true when $n=k+1$.

i.e. required to prove that $7^{k+1} - 3^{k+1} = 4P$, where $P \in \mathbb{Z}^+$

$$\begin{aligned} LHS &= 7^{k+1} - 3^{k+1} \\ &= 7.7^k - 3.3^k \\ &= 7\left(4M + 3^k\right) - 3.3^k \quad \text{by assumption} \\ &= 7 \times 4M + 7.3^k - 3.3^k \\ &= 7 \times 4M + 4.3^k \\ &= 4\left(7M + 3^k\right) \\ &= 4P, \quad \text{where } P \in \mathbb{Z}^+ \end{aligned}$$

$\therefore$ if $P(k)$ is true then $P(k+1)$ is true.

$\therefore$ by the process of mathematical induction $P(n)$ is true for $n \geq 1$.

13 (e) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Progress towards solution using the dot product	1

Sample Answer:

let $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{BC} = \underset{\sim}{b}$

since $ABCD$ is a parallelogram

$\overrightarrow{CD} = -\underset{\sim}{a}$ and $\overrightarrow{DA} = -\underset{\sim}{b}$

$\therefore \overrightarrow{AC} = \underset{\sim}{a} + \underset{\sim}{b}$ and $\overrightarrow{DB} = \underset{\sim}{a} - \underset{\sim}{b}$

to prove $\overrightarrow{AC}$ and $\overrightarrow{DB}$ are perpendicular we show $\overrightarrow{AC} \cdot \overrightarrow{DB} = 0$

$$\left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} - \underset{\sim}{b}\right) = \left|\underset{\sim}{a}\right|^2 - \left|\underset{\sim}{b}\right|^2$$

$$= 0 \qquad \text{since } AB = BC$$

$\therefore \overrightarrow{AC}$ and $\overrightarrow{DB}$ are perpendicular

Question 14 (15 marks)

14 (a)(i) (1 mark)
Outcomes assessed: ME12-4
Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

using the product rule:

$$\frac{d}{dx}\left(x\cos^{-1}x\right) = x \times \frac{-1}{\sqrt{1-x^2}} + 1 \times \cos^{-1}x$$

$$= \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

14 (a)(ii) (3 marks)
Outcomes assessed: ME12-4
Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correctly evaluates one integral	2
• Integration of all terms between $x = 0$ and $x = \frac{1}{2}$	1

Sample Answer:

integrating both sides of $\frac{d}{dx}\left(x\cos^{-1}x\right) = \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}$

$$\int_0^{\frac{1}{2}} \frac{d}{dx}\left(x\cos^{-1}x\right)dx = \int_0^{\frac{1}{2}} \cos^{-1}x\,dx - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\left[x\cos^{-1}x\right]_0^{\frac{1}{2}} = \int_0^{\frac{1}{2}} \cos^{-1}x\,dx - \int_0^{\frac{\pi}{6}} \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta \qquad \text{using } x = \sin\theta$$

$$\left[\frac{1}{2}\cos^{-1}\frac{1}{2} - 0\cos^{-1}0\right] = \int_0^{\frac{1}{2}} \cos^{-1}x\,dx - \int_0^{\frac{\pi}{6}} \frac{\sin\theta}{|\cos\theta|}\cos\theta\,d\theta$$

$$\frac{\pi}{6} = \int_0^{\frac{1}{2}} \cos^{-1}x\,dx - \int_0^{\frac{\pi}{6}} \sin\theta\,d\theta \qquad \sqrt{\cos^2\theta} > 0 \text{ since } 0 \le \theta \le \frac{\pi}{2}$$

$$\frac{\pi}{6} = \int_0^{\frac{1}{2}} \cos^{-1}x\,dx - \left(-\frac{\sqrt{3}}{2} + 1\right)$$

$$\therefore \int_0^{\frac{1}{2}} \cos^{-1}x\,dx = \frac{\pi}{6} + \left(1 - \frac{\sqrt{3}}{2}\right)$$

14 (b) (3 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correct integration	2
• Correctly separates the differential equation	1

Sample Answer:

$$\sec x \frac{dy}{dx} = \frac{e^{\sin x}}{y}$$

$$y\,dy = \cos x e^{\sin x}\,dx$$

integrating both sides:

$$\int y\,dy = \int \cos x e^{\sin x}\,dx$$

$$\frac{y^2}{2} = e^{\sin x} + c \qquad \text{since } \frac{d}{dx}\left(e^{\sin x}\right) = \cos x e^{\sin x}$$

given $x = 0$, $y = 0$

$$\frac{0^2}{2} = e^{\sin 0} + c$$

$$c = -1$$

$$\therefore y^2 = 2e^{\sin x} - 2$$

14 (c)(i) (2 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Attempts to let $\frac{dP}{dt} = 0$	1

Sample Answer:

The carrying capacity is found by letting $\frac{dP}{dt} = 0$

i.e. $P = 0$ or $5 - \frac{P}{10000} = 0$

$\therefore P = 50000$ is the carrying capacity.

The initial population of kangaroos is 15% of the carrying capacity.

$$\therefore P_0 = 0.15 \times 50000$$
$$= 7500$$

14 (c)(ii) (1 mark)

Outcomes assessed: ME11-4

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct solution	1

Sample Answer:

The rate of increase is a maximum when the population is half the carrying capacity.

$\therefore \frac{dP}{dt}$ is a maximum when $P = 25000$ kangaroos.

14 (d)(i) (2 marks)
Outcomes assessed: ME12-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Finds the correct value of μ or σ.	1

Sample Answer:

$$\hat{p} = \frac{6}{200}$$
$$= 0.03$$

$$\mu_{proportion} = \hat{p}$$
$$= 0.03$$

$$\sigma_{proportion} = \frac{\sigma}{n}$$
$$= \frac{\sqrt{np(1-p)}}{n}$$
$$= \frac{\sqrt{200 \times 0.03 \times 0.97}}{200}$$
$$= 0.01206 \text{ (5 decimal places)}$$

14 (d)(ii) (1 mark)
Outcomes assessed: ME12-5
Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

For 4 defective globes $\hat{p} = \frac{4}{200}$
$= 0.02$

$$z = \frac{0.02 - 0.03}{0.01206}$$
$= -0.829187...$
$= -0.83$ (2 decimal places)

14 (d)(iii) (2 marks)
Outcomes assessed: ME12-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Attempts use of the table of $P(Z < z)$ values	1

Sample Answer:

To find $P(4 < X < 5)$ we need to calculate the z – score of 5 defective globes

$$x = \frac{5}{200} = 0.025$$

$$z = \frac{0.025 - 0.03}{0.01206}$$
$= -0.41459...$
$= -0.41$ (2 decimal places)

$$\therefore P(4 < X < 5) = P(-0.83 < z < -0.41)$$
$$= P(z < -0.41) - P(z < -0.83)$$
$$= (1 - P(z < 0.41)) - (1 - P(z < 0.83))$$
$$= (1 - 0.6591) - (1 - 0.7967)$$
$$= 0.1376$$